The Joint Dynamics of Labour and Capital

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Abstract

Models of lumpy capital adjustment are too responsive to interest rates relative to empirical evidence. We argue that allowing for small convex adjustment costs in labour can help these models better match the data. Convex costs cause labour to increase slowly in response to a shock thus smoothing out the impact on the marginal product of capital. Due to both depreciation and uncertainty over future productivity, this delay in the benefits of additional capital can have a large impact on the responsiveness of capital investment.

1 INTRODUCTION

Firms make investment and hiring decisions jointly. When it is costly to adjust employment and there are frictions in investment, factor demands become dynamically interrelated. Does this interdependence matter for aggregate dynamics? Can it help resolve the tension between standard models of investment and the empirical evidence on capital adjustment at the firm/plant level?

This paper begins by documenting some new facts on the dynamic relationship between investment and changes in employment at the firm/establishment level. Using several datasets, we find that *lagged* employment growth is a statistically significant predictor of investment. Moreover, hiring/firing episodes increase the likelihood of upward/downward capital adjustments in the future.

We then argue that standard lumpy investment models have a hard time making sense of these findings. By treating labour as a flexible factor that can be maximized out of the production function, these models cannot explain why changes in employment predict investment. When the choice of labour is flexible firms time their hiring to coincide with investments in capital due to the complementarities in the production function. This combined with the low autocorrelation of investment in lumpy adjustment models means a high net hiring rate in the previous period predicts a low probability of making a large investment today. Indeed, we show that they generate a relationship between lagged employment growth and probability of capital adjustment that is opposite to the one we calculate from the data.

To address this, we introduce dynamic labour demand into a model of lumpy investment that allows for more complementarity in the production technology than the usual Cobb-Douglas-with-decreasing-returns-to-scale used in the literature. This addition helps provides a better fit to the data in several dimensions. First, it helps resolve the tension between large investments in the cross section and the relatively small empirical estimates of the elasticity of investment to interest rates. It rationalises the large investments as being made by firms with low capital to labour ratios. However, the magnitude of these large investments is less sensitive to interest rates than in the model with spot markets for labour. This is due to the greater convexity of the marginal return on capital as labour cannot be flexibly adjusted to fully benefit from any increases in investment. Additionally, the average elasticity is reduced by firms with high capital to labour ratios facing lower returns to investment due to the complementarities in production.

Second, the model can account for the predictive power of hiring decisions. Firms who anticipate they will invest soon, for example their capital stock is close to the lower bound (s,S) bound of the firm, will begin hiring in anticipation of the future higher marginal product of labour in order to raise their returns on capital investment.

A popular alternative to reduce the elasticity of investment has been to add convex costs in capital. We show that they imply implausibly long transitional dynamics following a permanent shock. While convex costs in labour do not imply such long transitional dynamics. Intuitively, the combination of convex and fixed costs in capital imply infrequent *and* small adjustments, leading to slow accumulation. On the other hand, with convex costs in labour firms continuously accumulate labour. When the infrequent capital adjustment occurs, the adjusments are larger because the marginal product of capital rises due to the accumulation of labour.

We use a novel perturbation approach to derive formulas for aggregate elasticities with dynamic interdependence and nontrivial firm heterogeneity. This allows us to shed light on how different forms of adjustment costs shape aggregate responses. Our headline result relates the time-0 extensive margin response of aggregate capital to the (weighted) second moment of investment in the cross section and an irreversibility parameter. We study the aggregate response to interest rate shocks and compute the dynamics triggered by permanent changes in corporate tax policy. We show that taking seriously the dynamic interdependence of capital and labour allows the model to match aggregate responses in the short and medium run. Finally, our model can generate positive autocorrelation in investment without relying on costs for adjusting the level of investment.

LITERATURE REVIEW

Existing work on factor adjustment at the firm level has focused on individual adjustment margins. Notable exceptions include Bloom (2009), Eslava et al. (2010), Sakellaris (2004) and Letterie et al. (2004). Eslava et al. (2010) build on the observation that shapes of adjustment hazards provide information about the nature of adjustment costs. If they are independent of gaps, they are consistent with quadratic adjustment costs. The fact that they find that employment and capital adjustment hazards depends on gaps suggests that firms do in fact face some form of lumpy adjustment costs. They also have strong evidence for irreversibility - capital destruction is much less likely than investment.

Our estimates of the dynamic response of the capital stock do not have implausibly long adjustment lags - a problem identified in Summers et al. (1981), who point out that q-theory delivers slow adjustment of the capital stock to changes in factor prices. The convex adjustment costs in capital used by Winberry (2021) imply extremely long transition dynamics following a permanent change in corporate tax policy. But recent micro level estimates imply more rapid adjustment of capital stock to the new steady state.

Bertola and Caballero (1994) argue that irreversibility at the firm level are important to understand aggregate investment dynamics whenever there is significant idiosyncratic uncertainty. More recently, Baley and Blanco (2022) characterize the macroeconomic effects of partial irreversibility in an environment without employment adjustment costs. Koby and Wolf (2020) show that if investment is sufficiently price elastic, general equilibrium smoothing makes the cross sectional distribution of capital irrelevant for aggregate investment dynamics. But they argue that large price elasticities are not consistent with the empirical evidence.

2 MODEL

Consider a discrete time economy with a continuum of firms who choose sequences of capital $\{k_t\}$ and labour $\{l_t\}$ to maximise the net present value of profits. They take as given the interest rate r and the wage w and face adjustment costs for both capital and labour, solving[1]

$$\max_{\{k_t,l_t\}_{t=1}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left[e_t F(k_t, l_t) - w l_t - AC(k_t, k_{t+1}, l_t, l_{t+1})\right]. \tag{1}$$

Idiosyncratic productivity $\{e_t\}$ follows an AR(1) with normal innovations. Capital depreciates at rate δ_k and there is labour attrition proportionate to δ_l. The production function F is CES with elasticity of substitution $\rho > 0$ and returns to scale parameter $\alpha \in (0,1)$. We allow for various forms of adjustment costs, so we let the adjustment cost function AC be given by

$$\begin{aligned} AC(k_t, k_{t+1}, l_t, l_{t+1}) = {} & \underbrace{w\xi}_{\textit{fixed cost}} \; \mathbb{1}_{\{k_{t+1} \neq (1-\delta_k)k_t\}} \\ & + \underbrace{p\left[1 - (1-\gamma)\mathbb{1}_{\{k_{t+1} \leq (1-\delta_k)k_t\}}\right]}_{\textit{partial irreversibility}} (k_{t+1} - (1-\delta_k)k) \\ & + \frac{\chi}{2} \underbrace{\left(\frac{k_{t+1} - (1-\delta_k)k_t}{k_t}\right)^2 k_t}_{\textit{variable cost capital}} + \frac{\phi}{2} \underbrace{\left(\frac{l_{t+1} - (1-\delta_l)l_t}{l_t}\right)^2 l_t}_{\textit{variable cost labour}}. \end{aligned}$$

The first adjustment cost is a fixed cost of capital adjustment which must be paid whenever tomorrow's capital stock is different from that which would be implied by depreciation. We assume that the fixed adjustment cost ξ is drawn at the start of each period from distribution G with mean μ. This adjustment cost captures the fact that output may be lost whenever new capital is installed.[2] The presence of this term is motivated by the literature on lumpy investment that has found this cost useful to replicate the long tail of firm investment observed in the data. As Caballero (1999) points out, this pattern of adjustment requires more than proportional adjustment costs. There has to be some benefit from incurring large adjustments and the simplest recipe is to introduce fixed costs. The mechanism by which it does so is that firms who receive a sequence of high fixed cost draws may choose to delay their investment till they receive a low draw at which point they will have a large gap between their capital and their target level of capital. The randomness plays a dual role: it helps computations by smoothing the problem and allows the model to generate more realistic investment distributions.

The second term represents partial irreversibility of capital and is motivated by evidence on the firm specificity of capital. To capture that specificity we allow a firm to buy capital at price p but assume it only receives γp whenever it decides to sell. The lost wedge $1-\gamma$ represents the share of the value of the capital

[1] We do not distinguish between employees and hours, i.e. we are assuming that hours are fixed.

[2] This may be because labour input has to be diverted to install new machines or because it is difficult to incorporate new machines into the production process.

that is firm specific. Irreversibility makes disinvesting very unattractive due to the low return on selling capital. Thus, when making a positive investment firms will reduce the size of it so as to lessen the loss if future shocks would leave them above their target capital. So irreversibility leads to a reduction in large investments and infrequent downward adjustments in capital.[3]

The third term captures variable costs that arise when adjusting the capital stock. This quadratic form was recently used by Winberry (2021) in order to generate investment to interest rate semi-elasticities that were closer to empirical evidence. The convex variable costs reduce the investment to interest rate semi-elasticity as they limit the size of investment any individual firm wants to make in any one period. Firms are instead incentivised to spread out their capital response to a shock over more periods in order to reduce the average cost per unit of investment. This solves the problem encountered in pure *Ss* models such as Khan and Thomas (2008) where firms who receive a small fixed cost draw make very large investments. At the same time, in the calibration used by Winberry (2021) the investment distribution is no longer skewed and there is no downward capital adjustment whenever there is some degree of irreversibility ($\gamma < 1$).

We depart from most of the literature on firm investment by introducing variable costs for changes in employment, the last term in the expression for AC. This quadratic form is somewhat standard in the dynamic labour demand literature and is a stand in for output loss that arises due to inexperience of new workers or severance payments that accrue whenever the firm decides to lay off workers. Although it is somewhat restrictive, it is consistent with labour hoarding, procyclical labour productivity and imperfect substitution between incumbent workers and new hires.[4] We also find that the empirical distribution of hiring/firing rates is roughly symmetric, making the use of this quadratic form somewhat less problematic.

The standard approach in the investment literature is to treat labour as a frictionless factor that can be *maximized out* of the production function. Incorporating this into the model makes employment decisions forward looking and slow-moving. In addition, it draws out the response of the marginal productivity of capital to productivity shocks, making investment less responsive. As we show below, this allows the model to better match the autocorrelation structure of investment. In addition, because labour is now forward looking the model can generate relationships like the ones we document in the empirical section.

To show this more formally, we now turn to our characterization of the firm's problem. We will use our recursive formulation of the problem to derive some formulas to understand the factors that determine the price-elasticity of investment.

[3]This is consistent with what we observe in the data- downward adjustments in capital are a lot less likely than upward adjustments. See Eslava et al. (2010)

[4]See Hamermesh (1993) and Mercan et al. (2021). This particular formulation makes the marginal cost of adjustment independent of the scale of the firm. Moreover, it implicitly assumes that adjustment costs depend on net rather than gross changes in employment. An alternative would be to let it depend on the change in the level of the input rather than the percentage change. At the end of the day, what cost structure is more realistic is an empirical question that this paper does not seek to address.

2.1 RECURSIVE CHARACTERIZATION

Let V denote the firm's value function. Given the state (e,k,l) and the realization of the fixed cost ξ, V tells the value of the maximized objective function (1). From the dynamic programming principle, V solves the following Bellman equation

$$V(e,k,l;\xi) = \max_{l',k'} \left\{ eF(k,l) - wl - AC(k,k',l,l';\xi) + \frac{1}{1+r}\mathbb{E}_{e'|e}\left[V^0(e',k',l')\right] \right\}, \tag{2}$$

where V^0 denotes the ex-ante value function (i.e. prior to the realization of the fixed cost)

$$V^0(e,k,l) \equiv \int V(e,k,l;\xi)G(d\xi). \tag{3}$$

We break up the maximization into two steps. First, given some value for productivity and employment today, we fix k' and find the optimal choice of employment going into next period. This defines an intermediate value function in terms of next period's capital stock and today's productivity and employment

$$\tilde{V}(e,k',l) \equiv \max_{l'} \left\{ \frac{1}{1+r}\mathbb{E}_{e'|e}\left[V^0(e',k',l')\right] - \frac{\phi}{2}\left(\frac{l'-(1-\delta_l)l}{l}\right)^2 l \right\}. \tag{4}$$

For future reference, we denote by $\tilde{\mathcal{L}}$ the employment policy that solves the problem associated with the intermediate value function $\tilde{V}$. Once we know the firm's optimal choice of capital going into next period, we can back out the firm's optimal employment policy by evaluating $\tilde{\mathcal{L}}$ at the optimal value of k'. It is convenient to work with $\tilde{V}$ because it makes the optimization problem for k' one-dimensional. To see this, note that given $\tilde{V}$ we can rewrite (2) as

$$\begin{aligned} V(e,k,l;\xi) = \max_{k'} \Bigg\{ & eF(k,l) - wl - w\xi\mathbb{1}_{\{k'\neq(1-\delta_k)k\}} - \frac{\chi}{2}\left(\frac{k'-(1-\delta_k)k}{k}\right)^2 k \\ & - p\left[1-(1-\gamma)\mathbb{1}_{\{k'\leq(1-\delta_k)k\}}\right]\left[k'-(1-\delta_k)k\right] + \tilde{V}(e,k',l) \Bigg\}. \end{aligned}$$

Going through the maximization over k', we arrive at

$$V(e,k,l;\xi) = eF(k,l) - wl + \max\left\{ V^{a,u}(e,k,l) - w\xi,\ V^{a,d}(e,k,l) - w\xi,\ \tilde{V}(e,(1-\delta_k)k,l) \right\} \tag{5}$$

with

$$V^{a,u}(e,k,l) \equiv \max_{k'\geq(1-\delta_k)k} \left\{ \tilde{V}(e,k',l) - \frac{\chi}{2}\left(\frac{k'-(1-\delta_k)k}{k}\right)^2 k - p\left[k'-(1-\delta_k)k\right] \right\}$$

and

$$V^{a,d}(e,k,l) \equiv \max_{k'\leq(1-\delta_k)k} \left\{ \tilde{V}(e,k',l) - \frac{\chi}{2}\left(\frac{k'-(1-\delta_k)k}{k}\right)^2 k - \gamma p\left[k'-(1-\delta_k)k\right] \right\}.$$

NO IRREVERSIBILITY, $\gamma = 1$

To understand the structure of the solution, let us first consider the case without irreversibility. In the absence of the price wedge, the Bellman equation becomes

$$V(e,k,l;\xi) = eF(k,l) - wl + \max\left\{ V^{a}(e,k,l) - w\xi,\ \tilde{V}(e,(1-\delta_k)k,l) \right\} \tag{6}$$

with

$$V^a(e,k,l) \equiv \max_{k'} \left\{ \tilde{V}(e,k',l) - \frac{\chi}{2} \left(\frac{k' - (1-\delta_k)k}{k} \right)^2 k - p \left[k' - (1-\delta_k)k \right] \right\}$$

From this, it is easy to see that the firm's decision on the extensive margin can be summarized by a cutoff rule according to which the firm adjusts its capital stock whenever

$$\xi \leq \frac{V^a(e,k,l) - \tilde{V}(e,(1-\delta_k)k,l)}{w} \equiv \hat{\xi}(e,k,l). \tag{7}$$

Combining this adjustment threshold with the cdf G, we obtain the state-dependent generalized hazard function of the model $\Lambda(e,k,l)$ that captures the adjustment probability given the state today:

$$\Lambda(e,k,l) \equiv G\left(\hat{\xi}(e,k,l)\right). \tag{8}$$

On the intensive margin, the firm chooses next period's capital stock in order to maximize the intermediate value function $\tilde{V}$ net of adjustment costs. For future reference, we denote by $\mathcal{K}$ the firm's optimal choice of capital conditional on adjustment

$$\mathcal{K}(e,k,l) \equiv \arg\max_{k'} \left\{ \tilde{V}(e,k',l) - \frac{\chi}{2} \left(\frac{k' - (1-\delta_k)k}{k} \right)^2 k - p \left[k' - (1-\delta_k)k \right] \right\}.$$

Integrating equation (6) with respect to the fixed cost ξ, using the definition of V^0 in (3) and the form of the investment policy described above, we arrive at an expression for the ex-ante value function without irreversibility:

$$\begin{aligned} V^0(e,k,l) = {} & eF(k,l) - wl + \Lambda(e,k,l)\left(V^a(e,k,l) - w\mathbb{E}\left[\xi | \xi \leq \hat{\xi}(e,k,l)\right]\right) \\ & + (1 - \Lambda(e,k,l))\tilde{V}(e,(1-\delta_k)k,l) \end{aligned} \tag{9}$$

This shows that the ex-ante continuation value of the firm is a weighted average of the value from adjusting both capital and labour optimally and the value from adjusting labour optimally and letting the capital stock depreciate. We now use this expression to derive necessary conditions for a firm's optimal choice of capital conditional on adjustment and the optimal employment policy. We will then use these optimality conditions to study the effects of a small exogenous change in the price of investment goods.

Optimal employment decision, $\gamma = 1$

From (9), the envelope condition for labour is

$$\frac{\partial V^0(e,k,l)}{\partial l} = eF_l(k,l) - w + \frac{\phi}{2} \left[\frac{\mathbb{E}_\xi \left[\mathcal{L}(e,k,l;\xi)^2 \right] - (1-\delta_l)^2 l^2}{l^2} \right], \tag{10}$$

with the *ex-post employment policy* $\mathcal{L}(\cdot;\xi)$ defined as

$$\mathcal{L}(e,k,l;\xi) \equiv \begin{cases} \tilde{\mathcal{L}}(e,(1-\delta_k)k,l), & \text{if } \xi > \hat{\xi}(e,k,l) \\ \tilde{\mathcal{L}}(e,\mathcal{K}(e,k,l),l), & \text{if } \xi \leq \hat{\xi}(e,k,l) \end{cases}.$$

In the absence of adjustment costs for labour, the marginal benefit is simply given by the marginal product. When $\phi > 0$, the marginal value of hiring an additional worker to the firm takes into account the effect on adjustment costs. Given this, the first order condition for employment is

$$\phi\left(\frac{l'-(1-\delta_l)l}{l}\right) = \frac{1}{1+r}\left(\mathbb{E}_{e'|e}\left[e'\right] F_l(k',l') - w + \frac{\phi}{2}\left[\frac{\mathbb{E}_{e'|e,\xi}\left[\mathcal{L}(e',k',l';\xi)^2\right] - (1-\delta_l)^2 l'^2}{l'^2}\right]\right). \tag{11}$$

When the firm chooses not to adjust the capital stock today, $k' = (1-\delta_k)k$ and the condition implicitly defines $\tilde{\mathcal{L}}(e,(1-\delta_k)k,l)$. When the firm chooses to adjust, $k' = \mathcal{K}(e,k,l)$ and (11) implicitly defines $\tilde{\mathcal{L}}(e,\mathcal{K}(e,k,l),l)$.

OPTIMAL INVESTMENT DECISION, $\gamma = 1$

The envelope condition for capital is

$$\frac{\partial V^0(e,k,l)}{\partial k} = eF_k(k,l) + (1-\delta_k)\left\{\Lambda(e,k,l)\ p + (1-\Lambda(e,k,l))\frac{1}{1+r}\mathbb{E}\left[\frac{\partial V^0(e',(1-\delta_k)k,\tilde{\mathcal{L}}(e,(1-\delta)k,l))}{\partial k}\right]\right\}$$

Note the fact that the envelope theorem does not apply when the firm chooses to let the capital stock depreciate and there is a direct effect of changing capital stock in those states of the world. Bringing back the explicit dependence on time, we can iterate this equation to derive the *sequence-space representation* for the marginal value of capital

$$\frac{\partial V^0(e_t,k_t,l_t)}{\partial k} = \mathbb{E}_t\sum_{j=0}^{\infty}\left(\frac{1-\delta_k}{1+r}\right)^j\left(\prod_{s=0}^{j-1}\left[1-G(\hat{\xi}_{t+s})\right]\right)\left(e_{t+j}F_{k,t+j} + G(\hat{\xi}_{t+j})p(1-\delta_k)\right) \tag{12}$$

Conditional on adjustment, the firm's choice optimal choice of capital is pinned down by the following optimality condition

$$p = \frac{1}{1+r}\times\mathbb{E}_t\sum_{j=0}^{\infty}\left(\frac{1-\delta_k}{1+r}\right)^j\left(\prod_{s=0}^{j-1}\left[1-G(\hat{\xi}_{t+1+s})\right]\right)\left(e_{t+1+j}F_{k,t+1+j} + G(\hat{\xi}_{t+1+j})p(1-\delta_k)\right)$$

We can also write the envelope condition for capital as

$$\frac{\partial V^0(e,k,l)}{\partial k} = eF_k(k,l) + (1-\delta_k)\mathbb{E}_\xi\left[Q(e,k,l;\xi)\right]$$

where $Q(\cdot;\xi)$ denotes *ex-post marginal value of capital*

$$Q(e,k,l;\xi) \equiv \begin{cases} \frac{1}{1+r}\mathbb{E}_{e'|e}\left[\frac{\partial V^0\left(e',(1-\delta_k)k,\tilde{\mathcal{L}}(e,(1-\delta_k)k,l)\right)}{\partial k}\right], & \text{if } \xi > \hat{\xi}(e,k,l) \\ p, & \text{if } \xi \leq \hat{\xi}(e,k,l) \end{cases}. \tag{13}$$

So an alternative way to write the FOC for capital is

$$p = \frac{1}{1+r}\left(\mathbb{E}_{e'|e}\left[e'\right]F_k(k',l') + (1-\delta_k)\mathbb{E}_{e'|e,\ \xi}\left[Q(e',k',l';\xi)\right]\right) \tag{14}$$

Partial irreversibility, $\gamma < 1$

When there is a wedge between the price of buying and the price of selling capital, the firm's investment policy changes. We now need to keep track of whether the firm is adjusting up or down.

Following the same logic as in the case without irreversibility, the firm's extensive margin decision can be summarized using two cutoff rules: adjust up whenever

$$\xi \leq \frac{V^{a,u}(e,k,l) - \tilde{V}(e,(1-\delta_k)k,l)}{w} \equiv \hat{\xi}^u(e,k,l) \tag{15}$$

and adjust down whenever

$$\xi \leq \frac{V^{a,d}(e,k,l) - \tilde{V}(e,(1-\delta_k)k,l)}{w} \equiv \hat{\xi}^d(e,k,l). \tag{16}$$

Combining these thresholds with the cdf G, we obtain a pair of state-dependent generalized hazard functions which we denote by Λ^u and Λ^d, respectively

$$\Lambda^u(e,k,l) \equiv G(\hat{\xi}^u(e,k,l)) \tag{17}$$

$$\Lambda^d(e,k,l) \equiv G(\hat{\xi}^d(e,k,l)) \tag{18}$$

On the intensive margin, the firm's optimal choice of capital when adjusting up maximizes the intermediate value function $\tilde{V}$ net of adjustment costs that apply whenever the firm chooses to increase capital. We denote by $\mathcal{K}^u$ the firm's optimal choice of capital conditional on upward adjustment

$$\mathcal{K}^u(e,k,l) \equiv \arg \max_{k' \geq (1-\delta_k)k} \left\{ \tilde{V}(e,k',l) - \frac{\chi}{2}\left(\frac{k'-(1-\delta_k)k}{k}\right)^2 k - p\left[k'-(1-\delta_k)k\right] \right\}.$$

Similarly, we let $\mathcal{K}^d$ denote the firm's optimal choice of capital conditional on downward adjustment

$$\mathcal{K}^d(e,k,l) \equiv \arg \max_{k' \leq (1-\delta_k)k} \left\{ \tilde{V}(e,k',l) - \frac{\chi}{2}\left(\frac{k'-(1-\delta_k)k}{k}\right)^2 k - p\,\gamma\left[k'-(1-\delta_k)k\right] \right\}.$$

Using the thresholds (15) and (16), we can integrate (5) to arrive at the following expression for the ex-ante value function

$$\begin{aligned} V^0(e,k,l) = {} & eF(k,l) - wl + \Lambda^u(e,k,l)\left(V^{a,u}(e,k,l) - w\mathbb{E}\left[\xi | \xi \leq \hat{\xi}^u(e,k,l)\right]\right) \\ & + \Lambda^d(e,k,l)\left(V^{a,d}(e,k,l) - w\mathbb{E}\left[\xi | \xi \leq \hat{\xi}^d(e,k,l)\right]\right) \\ & + \left(1 - \Lambda^u(e,k,l) - \Lambda^d(e,k,l)\right)\tilde{V}(e,(1-\delta_k)k,l) \end{aligned} \tag{19}$$

This expression relies on the fact that $\hat{\xi}^u(e,k,l) > 0 \implies \hat{\xi}^d(e,k,l) = 0$ and viceversa. This is because the firm is either to the left or to the right of it's target capital stock and thus never faces a non-zero probability of adjusting up and adjusting down within the same period (see Appendix … for a proof).

OPTIMAL EMPLOYMENT DECISION, $\gamma < 1$

The first order condition for employment is as in the case without irreversibility but with a slightly modified ex-post employment policy. This modification takes into account the fact that the target capital stock when adjusting up differs from the target when adjusting down. with the *ex-post employment policy* $\mathcal{L}(\cdot;\xi)$ now defined as

$$\mathcal{L}(e,k,l;\xi) \equiv \begin{cases} \tilde{\mathcal{L}}(e,(1-\delta_k)k,l), & \text{if } \xi > \max\left\{\hat{\xi}^u(e,k,l), \hat{\xi}^d(e,k,l)\right\} \\ \tilde{\mathcal{L}}(e,\mathcal{K}^u(e,k,l),l), & \text{if } \xi \leq \hat{\xi}^u(e,k,l) \\ \tilde{\mathcal{L}}(e,\mathcal{K}^d(e,k,l),l), & \text{if } \xi \leq \hat{\xi}^d(e,k,l) \end{cases}.$$

OPTIMAL INVESTMENT DECISION, $\gamma < 1$

We can also write the same first order condition for capital (14) as long as we modify the definition of $Q(\cdot;\xi)$ to take into account the difference in the ex-post marginal value of capital due to the difference between the price of buying and selling capital

$$Q^{irrev}(e,k,l;\xi) \equiv \begin{cases} \frac{1}{1+r}\mathbb{E}_{e'|e}\left[\frac{\partial V^0(e',(1-\delta_k)k,\mathcal{L}(e,k,l))}{\partial k}\right], & \text{if } \xi > \max\left\{\hat{\xi}^u(e,k,l), \hat{\xi}^d(e,k,l)\right\} \\ p, & \text{if } \xi \leq \hat{\xi}^u(e,k,l) \\ \gamma\, p, & \text{if } \xi \leq \hat{\xi}^d(e,k,l) \end{cases}. \tag{20}$$

3 AGGREGATE RESPONSES

The goal in this section is to understand what drives the response of firm aggregates to small changes in prices across different variants of the model laid out in Section 2. We first use a perturbation approach to characterize the time-0 elasticity of capital with respect to the price of investment goods. Then, we use an extended version of Auclert et al. (2019) to compute the dynamic responses with respect to all prices in the model.

3.1 TIME-0 RESPONSES

Our approach relies on equations (7), (8),(11) and (14) to compute the first order effect of a small change in the price of investment goods dp on firm aggregates. We denote by $d\mathcal{L}(x)$ the first-order response of the employment policy of a firm whose current state is given by $x = (e,k,l)$. Similarly, we let $d\mathcal{K}(x)$ denote the first-order response of the investment policy on the intensive margin. When there is no irreversibility, we let $d\Lambda(x)$ denote the first-order effect on the extensive margin. Whenever $\gamma < 1$, we use $d\Lambda^u(x)$ and $d\Lambda^d(x)$ for the first-order response of the pair of extensive margin hazards.[5]

[5] Formally, these are the Gateaux differentials of the optimal policies.

EXTENSIVE MARGIN, $\gamma = 1$

Let us begin with the price-elasticity of aggregate capital in the absence of irreversibility. The first-order time-0 response of aggregate capital $K \equiv \int \mathcal{K}(x;\xi)dD(x;\xi)$ is given by

$$dK = \int \underbrace{d\Lambda(x)\ \ [\mathcal{K}(x) - (1-\delta_k)k]}_{\text{extensive margin}} dD(x) + \int \underbrace{\Lambda(x)\ d\mathcal{K}(x)}_{\text{intensive margin}} dD(x) \tag{21}$$

A perturbation of (7) and (8) combined with the envelope theorem, establishes the following result on the extensive margin:

Proposition 1 *The first-order effect on the extensive margin $d\Lambda(x)$ is given by*

$$d\Lambda(x) = -\hat{g}(x) \times \frac{dp}{w} \times [\mathcal{K}(x) - (1-\delta_k)k]\,,$$

where $\hat{g}(x) \equiv g(\hat{\xi}(x))$. Therefore, the first-order time-0 response of aggregate capital due to the extensive margin is

$$dK^{ext} = -\frac{dp}{w} \times \int \hat{g}(x)\,[\mathcal{K}(x) - (1-\delta_k)k]^2\,dD(x).$$

This shows that the $\hat{g}$-weighted second moment of investment in the cross section is a sufficient statistic for the contemporaneous first-order effect due to the extensive margin. Much of the prior literature has used a uniform distribution leading $\hat{g}(x)$ to be constant. A uniform distribution estimated to generate the standard deviation of investment will imply that there are many firms on the margin of adjusting or not given their shock who could be induced to invest by a shock Thus leading to an extensive margin response much larger than has been estimated empirically. Alternative fixed cost distributions with support on the positive reals such as the log normal can help the model jointly match both the steady state distribution of investment and the response. Distributions such as the log normal will generate large investments from a sequence of high draws same as with the uniform, i.e. $\prod_{t=1}^{T}(1 - G(\text{Value of adjusting t periods after last adjustment}))$ is large enough. The difference is that for the log normal,$g(\text{Value of adjusting t periods after last adjustment})$ can be very small for t's that imply a large investment

Corollary 1 *The first-order time 0 response of aggregate labour due to the extensive margin is*

$$dL^{ext} = -\frac{dp}{w} \times \int \hat{g}(x)\,[\mathcal{K}(x) - (1-\delta_k)k]\,[\mathcal{L}^a(x) - \mathcal{L}^n(x)]\,dD(x).$$

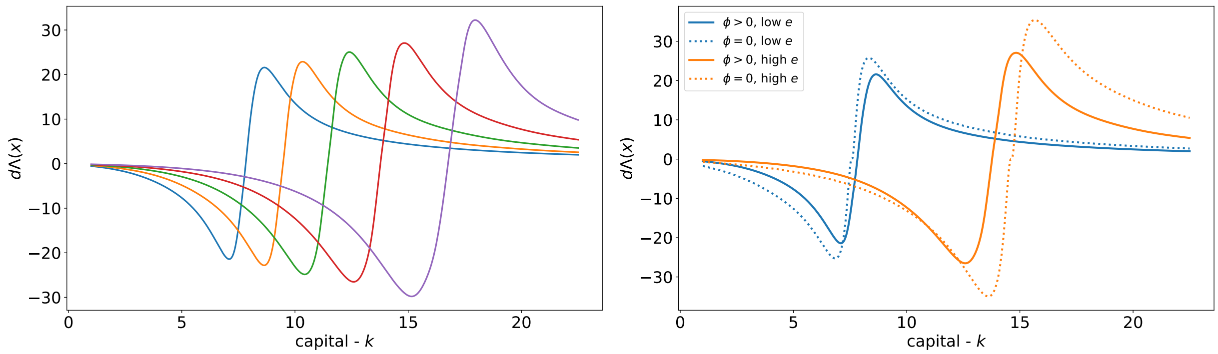
where $\mathcal{L}^a(x) \equiv \tilde{\mathcal{L}}(e, \mathcal{K}(x), l)$ and $\mathcal{L}^n(x) \equiv \tilde{\mathcal{L}}(e, (1-\delta_k)k, l)$. □

EXTENSIVE MARGIN, $\gamma < 1$

With partial irreversibility, the expression for the time-0 extensive margin response of aggregate capital becomes

$$dK^{ext} = \int d\Lambda^u(x)\,[\mathcal{K}^u(x) - (1-\delta_k)k]\,dD(x) + \int d\Lambda^d(x)\left[\mathcal{K}^d(x) - (1-\delta_k)k\right]dD(x) \tag{22}$$

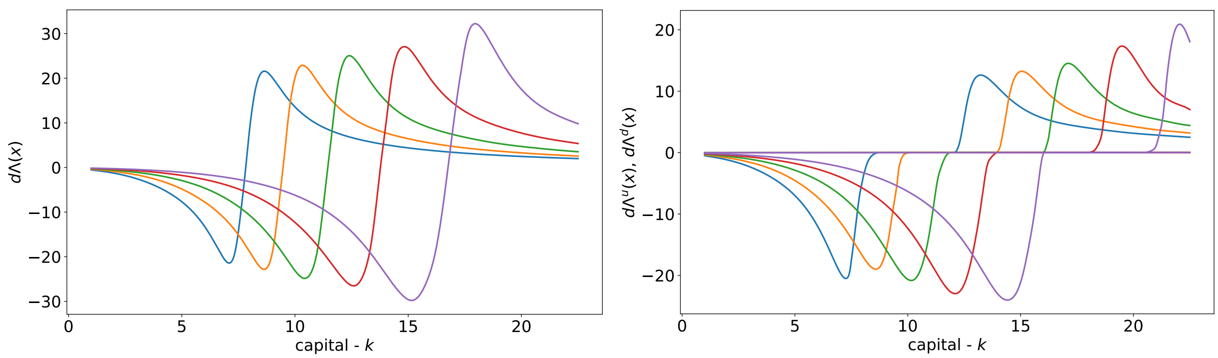
Using an argument similar to the one behind Proposition 1, we can establish the following result regarding the extensive margin response in the presence of irreversibility:



(a) $d\Lambda(x)$ along the k dimension

(b) $d\Lambda(x)$ with $\phi = 0$ and $\phi > 0$

Figure 1: Extensive margin responses without irreversibility



(a) $d\Lambda(x)$ along the k dimension

(b) $d\Lambda^u(x)$ and $d\Lambda^d(x)$ along the k dimension

Figure 2: Extensive margin responses with and without irreversibility

Proposition 2 *With irreversibility, the first-order effect on the extensive margin pair Λ^u and Λ^d is given by*

$$d\Lambda^u(x) = -\hat{g}^u(x) \times \frac{dp}{w} \times [\mathcal{K}^u(x) - (1-\delta_k)k]$$
$$d\Lambda^d(x) = -\hat{g}^d(x) \times \gamma\,\frac{dp}{w} \times \left[\mathcal{K}^d(x) - (1-\delta_k)k\right], \qquad \square$$

where $\hat{g}^u(x) \equiv g(\hat{\xi}^u(x))$ and $\hat{g}^d(x) \equiv g(\hat{\xi}^d(x))$. Therefore, the first-order response of aggregate capital due to the extensive margin is

$$dK^{ext} = -\frac{dp}{w} \times \left(\int \hat{g}^u(x)\,[\mathcal{K}^u(x) - (1-\delta_k)k]^2\,dD(x) + \gamma \int \hat{g}^d(x)\left[\mathcal{K}^d(x) - (1-\delta_k)k\right]^2 dD(x)\right).$$

With irreversibility, the sufficient statistics for the first-order effect on the extensive margin are the $\hat{g}$-weighted second moments of investment.[6]

[6]One also needs to take a stand on γ but maybe dK^{ext} could be the calibration target for γ?.We could target the intensive and extensive margin shares of the response and show how both K&T and Winberry miss the intensive margin. Can you hit them both with the things we have now?

Corollary 2 *With irreversibility, the first-order time 0 response of aggregate labour due to the extensive margin is*

$$dL^{ext} = -\frac{dp}{w} \times \Bigg\{ \int \hat{g}^{u}(x) \left[\mathcal{K}^{u}(x) - (1-\delta_k)k\right] \left[\mathcal{L}^{a,u}(x) - \mathcal{L}^{n}(x)\right] dD(x) + \gamma \int \hat{g}^{d}(x) \left[\mathcal{K}^{d}(x) - (1-\delta_k)k\right] \left[\mathcal{L}^{a,d}(x) - \mathcal{L}^{n}(x)\right] dD(x) \Bigg\}.$$

where $\mathcal{L}^{a,u}(x) \equiv \tilde{\mathcal{L}}(e, \mathcal{K}^{u}(x), l)$, $\mathcal{L}^{a,d}(x) \equiv \tilde{\mathcal{L}}(e, \mathcal{K}^{d}(x), l)$ *and* $\mathcal{L}^{n}(x) \equiv \tilde{\mathcal{L}}(e, (1-\delta_k)k, l)$. □

INTENSIVE MARGIN, $\gamma = 1$

We now turn to the characterization of the intensive margin response, which is somewhat more involved due to the interdependence of capital and labour. From (14), the perturbed FOC for capital is

$$p + \mu dp = \frac{1}{1+r}\left(\bar{e}(x)F_k\left(\mathcal{K} + \mu d\mathcal{K}, \mathcal{L} + \mu d\mathcal{L}\right) + (1-\delta_k)\left\{\mathbb{E}_{x'|x}\left[Q^0(e', \mathcal{K} + \mu d\mathcal{K}, \mathcal{L} + \mu d\mathcal{L})\right]\right\}\right)$$

with $\bar{e}(x) \equiv \mathbb{E}_{e'|x}[e']$ and $Q^0(x) \equiv \mathbb{E}_{\xi}[Q(x;\xi)]$. Taking a first-order Taylor expansion of this equation around the steady state and letting $\mu \to 0$, we find

$$d\mathcal{K}(x) = \frac{(1+r)\, dp - \left\{\bar{e}(x)F_{kl}(x) + (1-\delta_k)\mathcal{E}^{Q_l^0}(x)\right\} d\mathcal{L}(x)}{\bar{e}(x)F_{kk}(x) + (1-\delta_k)\mathcal{E}^{Q_k^0}(x)} \tag{23}$$

where $\mathcal{E}^{Q_l^0}(x) \equiv \mathbb{E}_{x'|x}\left[\frac{\partial}{\partial l}Q^0(x')\right]$ and $\mathcal{E}^{Q_k^0}(x) \equiv \mathbb{E}_{x'|x}\left[\frac{\partial}{\partial k}Q^0(x')\right]$ are the one-period ahead expected sensitivity of ex-ante marginal value of capital. To get some intuition, let us begin with the case where labour is fully flexible. In this case, the ex-post marginal value of capital Q does not depend on the firm's employment policy and the equation becomes

$$d\mathcal{K}(x) = \frac{(1+r)\, dp - \bar{e}(x)F_{kl}(x)d\mathcal{L}(x)}{\bar{e}(x)F_{kk}(x) + (1-\delta_k)\mathcal{E}^{Q_k^0}(x)}$$

where the change in Q^0 due to a small variation in the capital stock is given by

$$\frac{\partial}{\partial k}Q^0(x) = (1-\delta_k)\Bigg\{\hat{g}(x)\left(p - \frac{1}{1+r}\mathbb{E}_{e'|e}\left[\frac{\partial V^0\left(e', (1-\delta_k)k\right)}{\partial k}\right]\right)^2 + \left(\frac{1-\Lambda(x)}{1+r}\right)\mathbb{E}_{e'|e}\left[\frac{\partial^2 V^0\left(e', (1-\delta_k)k\right)}{\partial k^2}\right]\Bigg\}$$

After perturbing the FOC for labour when $\phi = 0$, we find

$$d\mathcal{L}(x) = -\frac{F_{lk}(x)}{F_{ll}(x)} \times d\mathcal{K}(x)$$

Substituting this into (23) and solving for $d\mathcal{K}(x)$ we can establish the following result for the intensive margin responses:

Proposition 3 *When* $\phi = 0$, *the intensive margin response* $d\mathcal{K}(x)$ *is given by*

$$d\mathcal{K}(x) = \frac{(1+r)\, dp}{\bar{e}(x)\left[F_{kk}(x) - \frac{F_{lk}(x)^2}{F_{ll}(x)}\right] + (1-\delta_k)\mathcal{E}^{Q_k^0}(x)}$$

Therefore, the first-order time-0 response of aggregate capital due to the intensive margin is

$$dK^{int} = (1+r)dp \times \int \frac{\Lambda(x)}{\bar{e}(x)\left[F_{kk}(x) - \frac{F_{lk}(x)^2}{F_{ll}(x)}\right] + (1-\delta_k)\mathcal{E}^{Q_k^0}(x)} dD(x)$$

Let us now consider the case where $\phi > 0$. The perturbed FOC for labour when the firm chooses not to adjust its capital stock is

$$\begin{aligned}(1+r)\phi\left(\frac{\mathcal{L}+\mu\hat{\mathcal{L}}-(1-\delta_l)l}{l}\right) &= \bar{e}(x)F_l\left((1-\delta_k)k, \mathcal{L}+\mu\hat{\mathcal{L}}\right) - w \\ &+ \frac{\phi}{2}\left[\frac{\mathbb{E}_{x'|x}\left[\mathcal{L}_2^0\left(e', (1-\delta_k)k, \mathcal{L}+\mu\hat{\mathcal{L}}\right)\right] - (1-\delta_l)^2\left(\mathcal{L}+\mu\hat{\mathcal{L}}\right)^2}{\left(\mathcal{L}+\mu\hat{\mathcal{L}}\right)^2}\right]\end{aligned} \tag{24}$$

where $\mathcal{L}_2^0(x) \equiv \mathbb{E}_\xi\left[\mathcal{L}(x;\xi)^2\right]$. This is the equation that applies for the employment policy of all those firms whose realization of the fixed cost $\xi > \hat{\xi}(e,k,l)$.[7] It is easy to see that the first order response of employment for the firms who choose not to adjust their capital stock when the shock hits is identically equal to zero. When the firm decides to adjust its capital stock, the perturbed FOC becomes

$$\begin{aligned}(1+r)\phi\left(\frac{\mathcal{L}+\mu\hat{\mathcal{L}}-(1-\delta_l)l}{l}\right) &= \bar{e}(x)F_l\left(\mathcal{K}+\mu\hat{\mathcal{K}}, \mathcal{L}+\mu\hat{\mathcal{L}}\right) - w \\ &+ \frac{\phi}{2}\left[\frac{\mathbb{E}_{x'|x}\left[\mathcal{L}_2^0\left(e', \mathcal{K}+\mu\hat{\mathcal{K}}, \mathcal{L}+\mu\hat{\mathcal{L}}\right)\right] - (1-\delta_l)^2\left(\mathcal{L}+\mu\hat{\mathcal{L}}\right)^2}{\left(\mathcal{L}+\mu\hat{\mathcal{L}}\right)^2}\right]\end{aligned} \tag{25}$$

Taking a first-order Taylor expansion of this equation and letting $\mu \to 0$, we find

$$\begin{aligned}&\left\{\left(\frac{(1+r)\phi}{l} - \bar{e}(x)F_{ll}(x)\right)\mathcal{L}(x) - \frac{\phi}{2}\left(\mathbb{E}_{x'|x}\left[\frac{\frac{\partial}{\partial l}\mathcal{L}_2^0(x')}{\mathcal{L}(x)}\right] - 2\,\mathbb{E}_{x'|x}\left[\frac{\mathcal{L}_2^0(x')}{\mathcal{L}(x)^2}\right]\right)\right\}d\mathcal{L}(x) \\ &= \left\{\bar{e}(x)F_{lk}(x)\mathcal{L}(x) + \frac{\phi}{2}\mathbb{E}_{x'|x}\left[\frac{\frac{\partial}{\partial k}\mathcal{L}_2^0(x')}{\mathcal{L}(x)}\right]\right\}d\mathcal{K}(x)\end{aligned} \tag{26}$$

Defining $\mathcal{E}^{\mathcal{L}_{2,l}^0}(x) \equiv \mathbb{E}_{x'|x}\left[\frac{\frac{\partial}{\partial l}\mathcal{L}_2^0(x')}{\mathcal{L}(x)}\right]$, $\mathcal{E}^{\mathcal{L}_{2,k}^0}(x) \equiv \mathbb{E}_{x'|x}\left[\frac{\frac{\partial}{\partial k}\mathcal{L}_2^0(x')}{\mathcal{L}(x)}\right]$, and $\mathcal{E}^{\mathcal{L}_2^0}(x) \equiv \mathbb{E}_{x'|x}\left[\frac{\mathcal{L}_2^0(x')}{\mathcal{L}(x)^2}\right]$ and solving for $d\mathcal{L}(x)$,

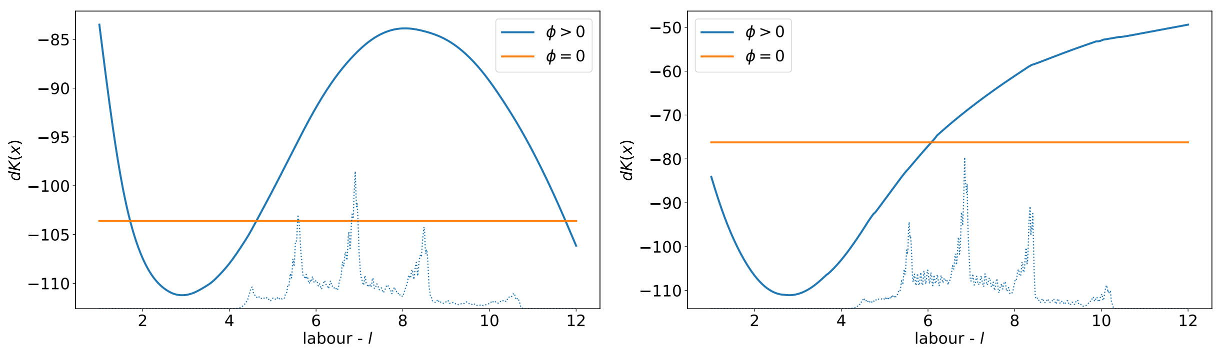
$$d\mathcal{L}(x) = \frac{\left\{\bar{e}(x)F_{lk}(x)\mathcal{L}(x) + \frac{\phi}{2}\mathcal{E}^{\mathcal{L}_{2,k}^0}(x)\right\}d\mathcal{K}(x)}{\left(\frac{(1+r)\phi}{l} - \bar{e}(x)F_{ll}(x)\right)\mathcal{L}(x) - \frac{\phi}{2}\left(\mathcal{E}^{\mathcal{L}_{2,l}^0}(x) - 2\,\mathcal{E}^{\mathcal{L}_2^0}(x)\right)} \tag{27}$$

Proposition 4 *When $\phi > 0$, the first-order responses $d\mathcal{K}(x)$ and $d\mathcal{L}(x)$ due to a small one-time change in the price of investment goods dp are given by the solution to the following system*

$$d\mathcal{K}(x) = \frac{(1+r)\,dp - \left\{\bar{e}(x)F_{kl}(x) + (1-\delta_k)\mathcal{E}^{Q_l^0}(x)\right\}d\mathcal{L}(x)}{\bar{e}(x)F_{kk}(x) + (1-\delta_k)\mathcal{E}^{Q_k^0}(x)} \tag{28}$$

$$d\mathcal{L}(x) = \frac{\left\{\bar{e}(x)F_{lk}(x)\mathcal{L}(x) + \frac{\phi}{2}\mathcal{E}^{\mathcal{L}_{2,k}^0}(x)\right\}d\mathcal{K}(x)}{\left(\frac{(1+r)\phi}{l} - \bar{e}(x)F_{ll}(x)\right)\mathcal{L}(x) - \frac{\phi}{2}\left(\mathcal{E}^{\mathcal{L}_{2,l}^0}(x) - 2\,\mathcal{E}^{\mathcal{L}_2^0}(x)\right)} \tag{29}$$

□



(a) $d\mathcal{K}(x)$ without irreversibility

(b) $d\mathcal{K}(x)$ with irreversibility

Figure 3: Intensive margin responses with and without irreversibility

The primary reason for the rise and then dip in the absolute value of the intensive margin response is the responsiveness of the marginal value of capital to additional capital first decreases and then increases. The initial decrease the responsiveness of the marginal value of capital to capital is driven by the fall in the effective DRS in capital as labour increases. Due to the quadratic adjustment costs on labour, when adjusting capital at a low level of labour, labour tomorrow will also be low and so the firm faces a high degree of DRS in capital causing it to not change its investment choice much in response to a change in capital price. But as labour rises this DRS lessens making firms for responsive. The later increase in the responsiveness of the marginal value of capital to capital is driven by the change in the derivative of the continuation value of capital in capital. If a firm is likely to adjust tomorrow additional capital will just increase or decrease the amount bought or sold tomorrow, thus the marginal value is constant. However if the firm is not likely to adjust then it will hold onto the additional value going forward so it the marginal value changes by the expected second derivative of the value function with respect to capital. Therefore as the probability of adjusting tomorrow falls the responsiveness of the marginal value of capital increases decreasing the intensive margin response.

3.2 Dynamic Reponses

In order to study the dynamic respones we use Auclert et al. (2019) to calculate the jacobians of the firm block of this model. We study the response of investment and labour to shocks to the real interest rate faced by firms. Then we look at the transitional dynamics following a persistent change in interest rates.

In 4 we plot the responses of investment and labour for different values of ρ, the elasticity of substitution in production. Reducing ρ even slightly below 1, i.e. Cobb-Douglas, has a large negative effect on the response of both labour and investment. The decline in responsiveness of both labour and capital is due to their complementarity in production. When firms reduce their investment by more the marginal product

[7]Note that we hold fixed tomorrow's employment policy because we are only considering a one-time shock that has no direct effect on future policies.

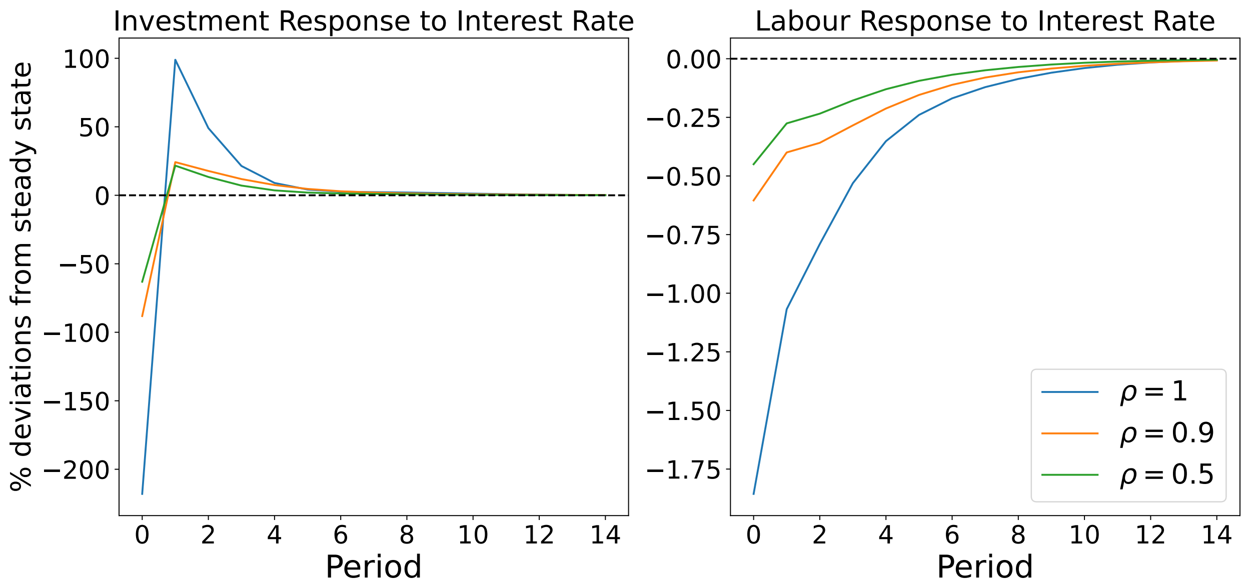


Figure 4: Impulse response to interest rate shock with varying substitutability

of labour declines leading them to hire less and vice versa. Recent evidence from Mark et al. (2021) based on change in bonus depreciation suggests that this elasticity may be far from 1.

In 5 we plot the responses of investment and labour for different values of ϕ, the parameter governing the size of labour adjustment costs. Similarly to the previous figure the responses for both investment and labour to an interest rate change move in the same direction as ϕ is changed. While unsurprisingly the effects on the labour response are large, the investment response declines by 25% going from a ϕ of 0 to one of 0.5. This further brings the model closer to empirical estimates of the responsiveness of investment to interest rates.

Finally in 6 we plot the response of our preferred parameterisation against the responses of the models of Winberry (2021) and Khan and Thomas (2008) following a persistent shock to interest rates. The shock takes the form of an AR(1) shock with a persistence of 0.9. Both of the alternative models display implausible dynamics. The model of Khan and Thomas (2008) features a huge response to the shock with capital falling by 25% almost immediately after the shock hits. In the model of Winberry (2021) the dynamics are highly drawn out. Capital remains below steady state even 50 quarters (12 years) after the shock at which point the shock is below 1% of its original size. When looking at the response to a permanent shock this feature is even more pronounced with the model of Winberry (2021) taking 25 yeras to reach the new steady state. The response of our model features both a initial response close to the data as well as a reasonable speed of recovery from the shock.

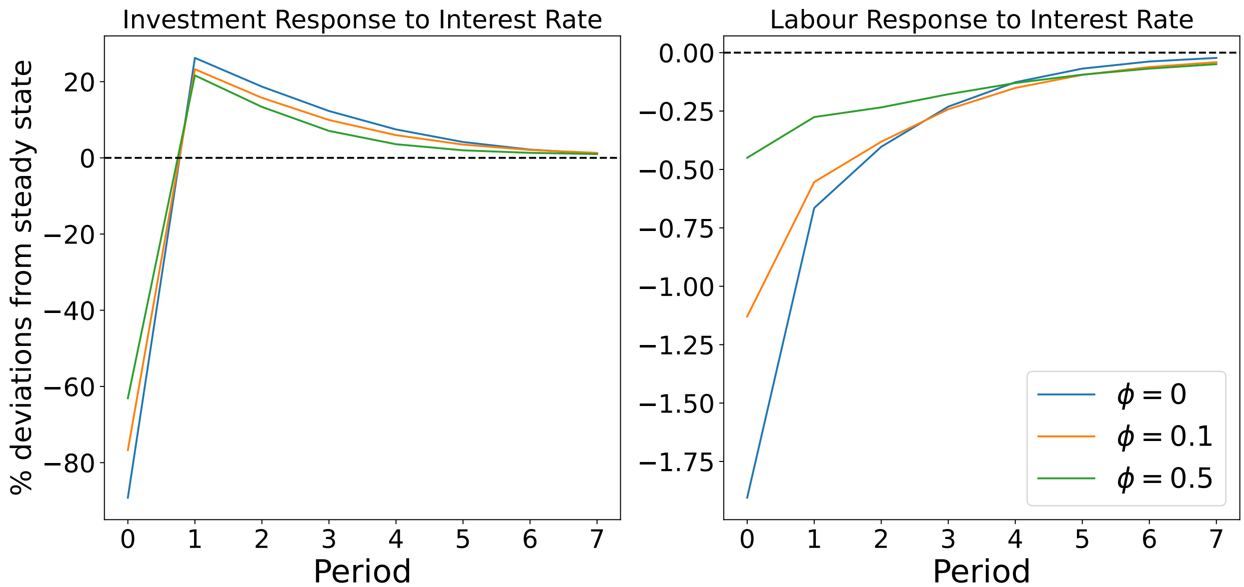


Figure 5: Impulse response to interest rate shock with varying labour adjustment costs

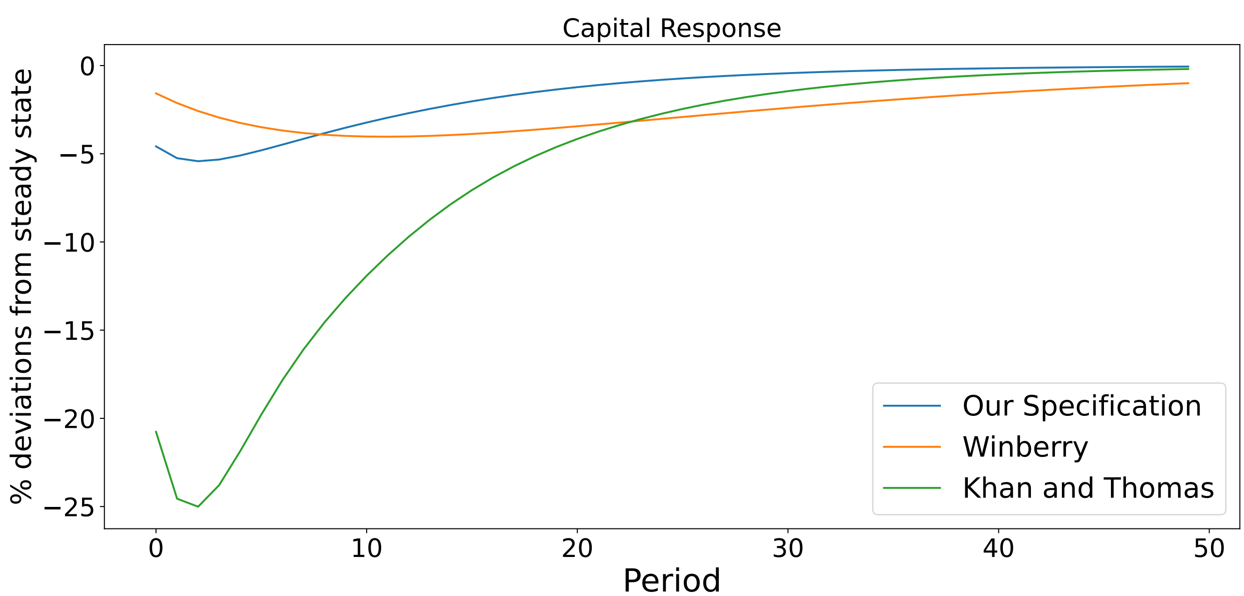
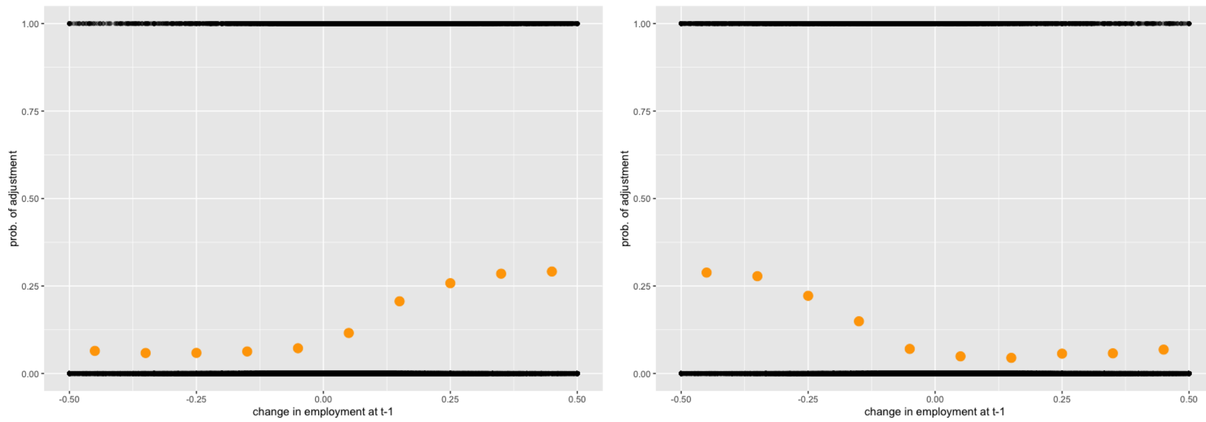


Figure 6: Impulse response of capital to permanent capital cost shock



(a) Prob. of adjusting up and lagged emp change

(b) Prob. of adjusting down and lagged emp change

Figure 7: Adjustment probabilities and lagged changes in employment with Compustat data

4 Measurement

4.1 Firm-level regressions

We start by building on the literature that documents the lagged investment effect. This is the result that lagged investment is a robust predictor of current investment even when controls for anticipated investment returns such as Tobin's Q and proxies for financial constraints such as the cash to assets ratio. For example, Eberly et al. (2012) argue that in terms of explanatory power, lagged investment is the best predictor of current investment.

We augment these specifications with lagged hiring to study the predictive power of employment growth for investment. The results are contained in Table 1. Consistent with the previous literature we find lagged investment is a statistically significant predictor of future investment. However comparing this coefficient to the magnitude of the

4.2 Inaction and hiring

In line with our findings in Section 4.1, Figure 7 shows that lagged changes in employment predict the likelihood of adjustment at the firm level. This is true for upward adjustments in both Chilean and Compustat data. The relationship with downward adjustments is present only in Compustat data.

4.3 Takeaways

We have argued that the data suggests employment is forward looking. We will now show that this relationship is not captured by models that treat labour as a frictionless factor that can be maximized out of the production function.

Dependent Variable:	real i/k					
Model:	(1)	(2)	(3)	(4)	(5)	(6)
Variables						
(Intercept)	0.0483***	0.0254***	0.0219***			
	(0.0007)	(0.0022)	(0.0018)			
real_i_k_lagged	0.1759***	0.1695***	0.1710***	0.0640***	0.1608***	0.0432***
	(0.0049)	(0.0049)	(0.0049)	(0.0050)	(0.0051)	(0.0053)
employment_change_lagged	0.1566***	0.1446***	0.1448***	0.1189***	0.1322***	0.1008***
	(0.0043)	(0.0044)	(0.0044)	(0.0046)	(0.0046)	(0.0048)
Q_lagged		0.0135***	0.0126***	0.0202***	0.0125***	0.0180***
		(0.0013)	(0.0013)	(0.0022)	(0.0014)	(0.0021)
cash_to_assets_lagged			0.0491***	0.2214***	0.1083***	0.2807***
			(0.0080)	(0.0120)	(0.0089)	(0.0135)
Fixed-effects						
gvkey				Yes		Yes
datadate-sic					Yes	Yes
Fit statistics						
Observations	132,729	132,729	132,729	132,729	132,729	132,729
R^2	0.08580	0.09819	0.09883	0.24135	0.24523	0.37415
Within R^2				0.05551	0.08771	0.04612

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table 1: Predicting investment

5 MODEL FIT

WORK IN PROGRESS

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A Continuous time model

In the continuous time version of the model, the firm solves a combined stochastic control and impulse control problem. When there is no adjustment, capital evolves as

$$dk_t = -\delta k_t dt + \sigma dB_t.$$

With impulse Δk, the capital stock jumpts from k to $k' = k + \Delta k$ but the firm has to pay a cost $\mathcal{G}$ that can be given by

$$\mathcal{G}(\Delta k) = F + p \max\{\Delta k, 0\} + p(1-\gamma) \min\{\Delta k, 0\}$$

As in the discrete time model, we assume that the firm faces convex and symmetric adjustment costs in labour. It can hire workers at rate n_t but must pay a cost $\mathcal{C}$ that is allowed to depend on current employment at the firm. A fraction ρ of workers leave the firm each period, so the evolution of labour is governed by

$$dl_t = [n_t - \rho l_t]\, dt$$

The program of the firm can be written as

$$v(k_0, l_0) = \sup_{\{\Delta k_t, n_t\}_{t \geq 0}} \mathbb{E}_0 \int_0^\infty e^{-rt} \left[eF(k_t, l_t) - wl_t - AC(\Delta k_t, n_t, k_t, l_t)\right] dt$$

subject to the evolution of capital and labour above and with

$$AC(\Delta k_t, n_t) := \mathcal{G}(\Delta k_t, k_t) + \mathcal{C}(n_t, l_t)$$