Skill-Biased Reallocation*

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Abstract

Workers displaced by the reallocation of labour demand across industries suffer persistent earnings losses, in a large part due to higher unemployment risk. This paper quantifies the aggregate unemployment implications of a reallocation of labour demand. I develop a search and matching model with multiple industries and industry specific skill that is calibrated to the US economy. In the model a reallocation shock leads to up to a 0.8 percentage points rise in unemployment. The combination of industry specific skill and the substitutability between workers of different skill levels are key to this result.

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1 Introduction

Many economic forces, such as automation and trade, cause the reallocation of labor demand across industries. I define a reallocation of labor demand as a change to the industry composition of employment that doesn’t change the long run level of aggregate unemployment. This leads workers in the shrinking industries to be displaced to other industries. A large literature has documented large worker level costs from reallocation in the form of earnings losses in part from higher unemployment risk. Recent papers on these earnings losses such as Huckfeldt (2022) and Traiberman (2019) have emphasized the importance of skill in explaining the losses. However, previous work on the impact of reallocation on aggregate unemployment has found no effect even when considering skill. Despite the movement of workers across industries leading to skill destruction if the skills of the workers are industry specific.

In this paper, I study how aggregate unemployment evolves along the transition in response to a reallocation of labor demand when skill is industry specific. I find that skills indeed matter, but a second essential factor is the degree of substitutability between workers with different levels of industry-specific skills. In a model with both of these features calibrated to the US economy, I find the typical magnitude of changes in industry employment shares over a decade can raise the unemployment rate by up to 0.8 percentage points.

The substitutability between workers with different levels of industry-specific skills is key as it determines how willing an industry is to hire incoming work-

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1See Davis and Von Wachter (2011), Autor, Dorn and Hanson (2016), Neal (1995) and Walker (2013) for some examples across different topics.
ers. When a reallocation of labour demand occurs between two industries there is a net movement of workers to the growing industry. As the entering workers cannot transfer their industry specific skills to the industry they enter as unskilled. Thus the supply of unskilled workers in the growing industry increases but not the supply of skilled workers in the short run. Due to the complementary in production, this increase in relative supply causes the marginal product of the entering workers to decline. Thus firms in the industry are not willing to hire all the incoming workers, leading to unemployment. In the long run as the workers who moved develop industry specific skill, unemployment returns to its steady state level.

To assess the quantitative importance of this mechanism I build a quantitative search and matching model with multiple industries. Workers accumulate industry-specific skill while employed in a stochastic manner. If a worker switches industries, they lose their accumulated skill. This acts as a mobility friction as workers who have accumulated skill are less likely to move as they would lose the wage premium associated with their accumulated skill. Then, instead of assuming perfect substitutability, I assume the industry-level production function has constant elasticity of substitution over workers of different skill levels. A low elasticity of substitution corresponds with the case where skilled workers are doing different and complementary work to that of unskilled workers. Additionally, I allow firms to direct their vacancies by the skill level of the workers.

To calibrate the model, I use heterogeneity in the observed returns to industry tenure and transition probabilities across industries. I then validate
this calibration by comparing earnings losses of displaced workers to those estimated by Huckfeldt (2022). The model estimates match the on impact decline in earnings as well as the dynamics for displaced workers reemployed in the same industry and those not.

Then, I use the quantitative model to assess the impact of reallocation of labor demand on aggregate unemployment. I formally model the reallocation as being caused by a shock that raises productivity in one industry and lowers it in another. The magnitude of the productivity shocks is set to match the average decadal dispersion in industry employment share growth rates as well as to keep steady state unemployment constant. I find the shock leads to a rise in unemployment of up to 0.8 percentage points. Additionally, there is a large amount of heterogeneity in the impact of reallocation. When the reallocation is towards industries for which industry specific skill is less important the magnitude of the rise in unemployment is only 0.2 percentage points.

The elasticity of substitution between workers of different skill levels in production is a key determinant of the magnitude of the rise in unemployment. Taking this elasticity to infinity which is the case of perfect substitutability, the effect of reallocation on unemployment becomes negligible. As the elasticity of substitution increases, the marginal product of workers of different skill levels is less dependent on the relative employment of workers of different skill levels. Thus when unskilled workers move to the growing industry the firm is willing to hire more of them as their marginal product declines only a little. Taking the elasticity of substitution from 0.5 to 2 leads to over a 50% reduction in the level of transitory unemployment caused by the shock.
Sahin et al. (2014) propose a measure of the degree of unemployment that occurs due to a mismatch between unemployed workers and vacancies. Replicating their measure in the model, it attributes only 1% of the peak of unemployment to mismatch. This is because unemployed workers are not mismatched in the sense that they are not searching for jobs in the wrong industry. Instead, they don’t have the skills that firms are posting vacancies for. Extending the measure of mismatch to include skill increases the share of unemployment due to mismatch to 17.5% at the peak. Here mismatch is occurring within industry rather than across industries.

I then consider how the results change if the assumption of directed search over skill is replaced with random search. Calibrating the random search model to the same moments as the directed search model, I find no large rise in unemployment in response to the same shock. This is because the effect of the marginal products changing cancel out in the vacancy posting decision. As the value of unskilled workers declines, the value of skilled workers increases thus leaving the value of lottery between them unchanged. Additionally, the distribution of skill among the unemployed converges quickly to the overall distribution in the industry as there is no selection in which workers are hired or separated.

Literature Review The finding of reallocation causing aggregate unemployment builds upon the literature on the worker level costs of reallocation. Neal (1995) showed that among workers who switched industries those with higher tenure suffered larger earnings losses. Walker (2013) showed workers
exposed to increases in production costs due to the clean air act experienced persistent earnings losses over time which in part driven by increased nonemployment. Davis and Von Wachter (2011) and Jarosch (2023) find large persistent earnings losses for workers displaced from their jobs. Huckfeldt (2022) argues that hiring becoming more selective in recessions can explain the increase in earnings losses from displacement during recessions. This paper takes elements such as specific skills and directed search but shows that reallocation can have aggregate unemployment effects.

Many papers starting with Lilien (1982) and Rogerson (1987) but also including Dvorkin (2014), Pilossoph (2012), Chodorow-Reich and Wieland (2020) and Carrillo-Tudela and Visschers (2023) studying the aggregate effects of reallocation. These papers all use search and matching models with multiple sectors to study the impact of reallocation on unemployment. I contribute to this literature in two ways, the first contribution concerns the substitutability between workers of different skill levels. While the previous literature has assumed perfect substitutability, in this paper I allow for imperfect substitutability. Kambourov and Manovskii (2009) contains a model with occupational-specific skill and a CES sectoral production function but does not consider the impact of reallocation on unemployment. Instead, it focuses on the link between occupational mobility and wage inequality. I show that relaxing this assumption has a large impact on the effect of reallocation on unemployment. For estimates of this substitutability in the range of the empirical literature, the effect of reallocation on unemployment is quantitatively sizable. This is unlike the null results found in Pilossoph (2012), Carrillo-Tudela and Visschers
Mercan, Schoefer and Sedláček (2024) make a related assumption that newly hired workers are imperfectly substitutable with incumbent workers in the initial period they are hired. There are two major differences. First, workers take longer to become skilled than in their model in which it takes a quarter. Secondly, in this paper workers retain their skill if they remain within the industry. Thus a separation shock would not have a large effect as the skilled workers would be quickly rehired. However, if the separations shocks they identify are driven by shocks that also cause reallocation then there would be a slow recovery of employment in the model from this paper.

The second contribution is I allow for heterogeneity in the importance of industry-specific skill across industries. Both Carrillo-Tudela and Visschers (2023) and Kambourov (2009) allow for occupational-specific skills but don’t allow the accumulation process to differ across sectors. Wiczer (2015) allows the skill level that workers who have just entered an occupation have relative to higher tenure workers to differ across occupations. The speed of accumulation is fixed, however, at one model period limiting the degree of heterogeneity. By allowing for heterogeneity in the importance of industry-specific skill I find reallocations of the same magnitude can have very different effects on unemployment depending on the industries affected.

This paper is also related to the literature on mismatch unemployment. Shimer (2007) and Şahin et al. (2014) study how mismatch between workers

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2Chodorow-Reich and Wieland (2020) argue that reallocation only causes a rise in unemployment during recessions. This is because reallocation exacerbates the binding of nominal wage rigidity. They find however no effect outside of a recession when nominal wage rigidity isn’t binding.
and the industry they are searching in can lead to unemployment. This paper highlights how there can be mismatch between skills demanded and supplied within an industry not just across industries.

This paper also relates to the literature on the impact of trade shocks when there are costs to switching sectors of which Artuç, Chaudhuri and McLaren (2010) is a seminal paper. The contribution of this paper is to show that trade shocks can lead to transitory unemployment as the economy adjusts. Traiberman (2019) highlights the importance of specific skills for explaining the distribution of income responses to a trade shock, however, they don’t consider the impact on unemployment. Caliendo, Dvorkin and Parro (2019) was among the first papers to study the dynamic unemployment response in a general equilibrium model with frictions to switching sector. However, they model unemployment as a choice in a Roy style model and so acts as insurance and dampens the welfare impact of the trade shock. Kim and Vogel (2020) and Galle, Rodríguez-Clare and Yi (2023) study unemployment due to downward nominal wage rigidities using comparative statics. Dix-Carneiro (2014) features a CES production function over fixed skill types in a multi-industry model where unemployment is an sector which can be chosen. It does have specific skill accumulation but different levels of skill along that dimension are perfectly substitutable. Dix-Carneiro et al. (2023) allows for frictional unemployment and studies the dynamics of unemployment due to trade shocks. In their model, frictions to switching sector slow the adjustment of the economy but do not lead to transitory unemployment.\footnote{The changes in unemployment are driven by two forces. First, unemployment falls to the overall gains from trade. Second, they calibrate different industries to have different}
Another place the analysis in the paper applies to is the case of automation. Eden and Gaggl (2018) and Vom Lehn (2020) study the effects of automation with substitutability between routine and non-routine workers but don’t allow for unemployment. Humlum (2019) studies robot adoption in a model with multiple occupations, occupation specific skill, however the focus is on the distribution of earnings and there is no unemployment. Jaimovich et al. (2021) extends these analyses to allow for unemployment but only as a form of occupational choice. Restrepo (2015) studied unemployment due to automation in a search model with frictional unemployment with skilled and unskilled workers. The driving force of unemployment in his model is search being undirected. When there are more unskilled workers firms post fewer vacancies as an expected match is less productive. In this paper instead, the search is directed but the marginal product of unskilled workers is lower when the relative supply of them is higher.

2 Illustrative Framework

I first consider a simple discrete time one industry model to illustrate how imperfect substitutability combined with industry specific skills can lead to short run unemployment in response to reallocation. The one industry being modelled is the one whose labor demand increases in response to reallocation. This is because the effect of the decline in labor demand on unemployment rates so as the relative size of industries fluctuates the unemployment rate changes. Unemployment in the US rises initially as production shifts towards manufacturing temporarily. In the appendix they show that frictions to mobility actually lessens this rise by reducing the movement towards manufacturing in the short.
in the shrinking industry is consistent with many models of labor markets. What will determine the aggregate impact is how unemployment responds in the expanding industry.

I assume that aggregate output is a function of two labor inputs, one produced by workers with low industry specific skill \((N)\) and one produced by workers with high industry specific skill \((S)\). I will refer to these workers as unskilled or skilled respectively. There is also a neutral productivity level \(A\) and so the production function can be written

\[
Y = AF(N, S)
\]

To simplify the analysis I assume that all the skilled workers are always employed. On the other hand the employment of unskilled workers is determined by search and matching. Firms must post vacancies \(v\) at cost \(\kappa\) in order to hire workers. A worker firm match is only productive the period after the match forms. The number of matches is determined by a matching function \(m(v, u)\) that is constant returns to scale in the number of unemployed workers \(u\) and the number of vacancies. Then the vacancy fill rate \(q(v, u) = \frac{m(v, u)}{v}\) and the job finding rate \(f(v, u) = \frac{m(v, u)}{u}\) can be defined. The matches that workers form with firms exogenously separate after production each period with probability \(\delta\). In steady state the number of separations must equal the number of new matches formed. Denoting the size of the unskilled labor force as \(L\)
\[ \delta(L - u) = f\left(\frac{v}{u}\right)u \]

This equation then pins down the level of unemployment as a function of market tightness \( \theta = \frac{v}{u} \) the ratio between vacancies and unemployed workers.

\[
\frac{u}{L} = \frac{\delta}{\delta + f(\theta)} \tag{1}
\]

The level of vacancy posting is determined by a free entry condition. Vacancies are posted until the value of a posted vacancy is equal to the cost of posting it. The value of a vacancy is equal to the vacancy fill rate times the expected value of a match to the firm. The expected value of a match to the firm is the expected discounted profit. For simplicity I assume the wage is constant, and the output of the match is the marginal product of the worker \( F_N \). Thus the expected discounted profit is

\[ \frac{\beta(F_N - w)}{1 - \beta \delta} \]

Firms discount at rate \( \beta \delta \) as matches may break up with probability \( \delta \) and \( \beta \) is the discount rate. Thus the equilibrium is determined by the system of two equations in \( \theta \) and \( u \)
\[
\frac{u}{L} = \frac{\delta}{\delta + f(\theta)} \quad (2)
\]

\[
\frac{\kappa}{q(\theta)} = \frac{\beta(F_N - w)}{1 - \beta \delta} \quad (3)
\]

Where the left hand side of Equation 3 is the marginal cost of hiring a worker and the right hand side is the marginal profit.

I will consider two cases, first the case of perfect substitutability between unskilled and skilled workers i.e. \( F_{NN} = 0 \). Second the case of imperfect substitutability i.e. \( F_{NN} < 0 \). In Figure 1 I plot the initial steady state equilibrium for both cases where I choose the equilibrium level of employment to be the same. The marginal hiring cost curve is convex and upwards sloping because \( q(\theta) \) is convex in \( \theta \) and the employment level is increasing in \( \theta \). For perfect substitutability the marginal profit curve is flat while in the case of imperfect substitutability it is downwards sloping.

I then add two shocks to the model to mimic the effect of reallocation on the expanding industry. The first shock is a positive shock to \( A \), this is the direct effect of the shock causing reallocation. It raises the marginal products of workers in the expanding industry thereby increasing demand for them. The second shock is an entry of unskilled workers so that \( L \) goes up. This mimics the indirect effects of reallocation on the expanding industry. As workers in the shrinking industry face lower wages and higher unemployment they move to the expanding industry. That these entering workers are unskilled because the skill under consideration here is industry specific so even if they were skilled
in their previous industry in the expanding industry they are unskilled.

I plot the impact of these shocks in Figure 2. The positive shock to $A$ raises the marginal profit curve in both cases. While the entry shock shifts the marginal hiring cost curve down. This is because the larger pool of workers mean firms have to post fewer vacancies to maintain the same level of employment.

As can be seen in Figure 2 the new steady state equilibrium under imperfect substitutability features lower employment than under perfect substitutability. This is because as the entering workers become hired the marginal product of unskilled workers declines as the number of skilled workers is fixed. Thus
the marginal profit of new workers declines leading to firms to let the market
tightness slacken and therefore a smaller employment response. Since there is
a negative employment effect in the unmodelled shrinking industry this smaller
positive employment response leads to an aggregate rise in unemployment.

Not only is the employment response smaller but depending on the degree
of imperfect substitutability the unemployment rate in the expanding industry
can rise in response to reallocation. In Figure 2 the new equilibrium under
imperfect substitutability occurs at a lower value of $\theta$ than in the initial equi-
librium. This can be seen from the marginal hiring cost curve being intersected
at a lower level in the y axis. As discussed above in Equation 1 the unem-
ployment rate is a function of $\theta$ and a lower $\theta$ implies a higher unemployment
rate.

This comparison of steady state equilibria is short run in the sense that
the supply of skilled workers is fixed. In the long run this supply adjusts such
that the employment level in both substitutability cases is equal. So in the
long run there is no unemployment effect from the shock.

In the rest of the paper I will build and calibrate a dynamic general equi-
librium model featuring industry specific skills and imperfect substitutability
between workers with different levels of these skills. The model will allow me
to quantify the magnitude of the mechanism discussed in this section. Addition-
ally, I will use the model to study the dynamic response of unemployment
to reallocation.
3 Quantitative Model

I build a search and matching model in which workers can switch industries if separated. Following Artuç, Chaudhuri and McLaren (2010) I model this switching decision as a discrete choice subject to taste shocks. The main contrast of this model from the literature is allowing for the marginal product of a worker to depend on the distribution of skill within the industry.
3.1 Labor Market

There is a separate labor market for each industry \((k)\) - skill level\((s)\) pair. Firms can post vacancies \(v(k, s)\) in the market of their choosing. There is also a pool of unemployed workers \(u(k, s)\) for each worker skill - industry pair. The market tightness for a given labor market is defined as usual as vacancies divided by unemployed workers \(\theta(k, s) = \frac{v(k, s)}{u(k, s)}\). The labor markets have a matching friction in the form of the standard cobb-douglas matching function

\[
m(u, v) = \mu u^\rho v^{1-\rho}
\]

Where \(\rho\) is the elasticity of matching and \(\mu\) is a matching efficiency parameter. The cobb-douglas matching function can produce more matches than either the number of unemployed workers or vacancies. In these cases, I truncate the number of matches to the minimum of the number of unemployed workers or vacancies. Given this matching function and the definition of labor market tightness, the job finding rate can be written \(f(\theta(k, s)) = \frac{m(u(k, s), v(k, s))}{u(k, s)} = \theta(k, s)^{1-\rho}\). The vacancy fill rate can similarly be written as \(q(\theta(k, s)) = \theta^{-\rho}\).

3.2 Workers

There is a unit mass of workers, who are risk neutral and discount at rate \(\beta\). They can either be employed or unemployed and are at all times attached to an industry. Workers employed in an industry at the start of a period keep their job with a fixed probability \((1 - \delta)\) and lose it with probability \(\delta\). This
timing is based on Christiano, Eichenbaum and Trabandt (2016). It allows for the possibility of workers switching jobs without a period of unemployment consistent with the large numbers of job-to-job transitions observed in the data. Those that lose their job at the beginning of the period or who were unemployed at the start of the period face a choice over whether to change industries. I model this as a discrete choice where workers choose the sector $k'$ that maximises their utility

$$S_t(k, s, \zeta) = \max\{U_t(k, s) + \zeta_{i,0}, \max_{k' \neq k} U_t(k', 0) - \alpha_{k,k'} + \zeta_{i,k'}\}$$

Where $U(k', 0)$ is the expected utility from being in sector $k'$ with no industry-specific skill, $\alpha_{k,k'}$ is a utility cost of switching from sector $k, k'$ and $\zeta_{i,k'}$ is the type 1 extreme value taste shock for sector $k'$ which is iid across sectors and time and has variance $\sigma_\zeta$. The type 1 extreme value taste shocks generate a motive for gross moves. Some workers in industry $k$ will draw a high taste shock for industry $k'$ and so will want to switch to that industry and vice versa. Additionally the shocks and mean that the probability of switching from $k$ to $k'$ can be written tractably as

$$P(k \rightarrow k'|s) = \frac{e^{(U_t(k',0) - \alpha_{k,k'})/\sigma_\zeta}}{\sum_{k'=k} e^{(U_t(k,0) - \alpha_{k,k})/\sigma_\zeta}}$$

The expected value function when making the choice has the following form
\[ S_t(k, s) = E_\zeta[S_t(k, s, \zeta)] = \sigma_\zeta(\gamma + \log(e^{U_t(k, s)/\sigma_\zeta} \sum_{k! = k} + e^{(U_t(\hat{k}, \theta) - \alpha_k \hat{k})/\sigma_\zeta})) \]

I add a search cost of \( \sigma_\zeta \gamma + \sigma_\zeta \log(n_k) \) to eliminate most of the gains in utility from search due to the type 1 extreme value shocks. The first part \( \sigma_\zeta \gamma \) reflects the mean type I extreme value while \( \sigma_\zeta \log(n_k) \) eliminates the gains due to more alternatives which increase the expected value of the maximum shock. This ensures workers don’t prefer to be unemployed in order to be exposed to the taste shocks.

Once industry switching decisions are made, unemployed workers search for a job in the job market associated with their current industry and level of skill human capital. They thus find a job with probability \( f(\theta(k, s)) \) and remain unemployed with probability \( 1 - f(\theta(k, s)) \). After this production occurs, the employed receive a wage \( w(k, s) \) and the unemployed receive unemployment benefits \( b \). Finally, at the end of the period, two events can occur. First employed workers potentially gain human capital in their current industry with probability \( \psi_k \). On the other hand, unemployed workers lose their industry-specific human capital with probability \( \rho \). The second event is that a proportion \( d \) of workers die and are replaced by unemployed workers in the same industry with no industry-specific skill. I add death to the model as I will target wage growth in calibrating the human capital parameters. As workers experience wage growth over the lifecycle, not adding death will lead to too many workers with human capital in the steady state distribution. Given this
the values of employment $V$ and unemployment $U$ are

$$V_t(k, s) = \delta S_t(k) + (1 - \delta)(w_t(k)$$
$$+ m_t (1 - d)((1 - \psi(k))V_{t+1}(k, s) + \psi(k)V_{t+1}(k, s + 1)))$$

$$U_t(k, s) = f(\theta_t(k))(w_t(k) + m_t (1 - d)((1 - \psi(k))V_{t+1}(k, s) + \psi(k)V_{t+1}(k, s + 1)))$$
$$+ (1 - f(\theta_t(k)))(b + m_t (1 - d)((1 - \rho(k))S_{t+1}(k, s) + \rho(k)S_{t+1}(k, s - 1)))$$

3.3 Firms

There is a continuum of firms in each industry which each employs one worker. A firm must post a vacancy in order to hire a worker. The cost of posting a vacancy for a worker of skill $s$ is denoted $\kappa(k, s)$ and there is free entry into the market for vacancies. This implies the free entry condition for firms

$$\kappa = q(\theta)E[J_t(k, s)]$$

Where $J_t(k, s)$ is the value of a filled vacancy, which solves the following Bellman equation.

$$J_t(k, s) = (y(k, s) - w(k, s)) + \beta(1 - d)(1 - \delta)[(1 - \psi(k))J_{t+1}(k, s) + \psi(k)J_{t+1}(k, s + 1)]$$

Where $y(k, s)$ is the revenue generated by a match of worker with skill $s$
in industry $k$. The interpretation of $\kappa$ is of effective vacancy posting cost as the productivity of the matching function will not be separately pinned down in the calibration.

I assume wages are set by Nash bargaining between the firm and worker with equal bargaining weights. So in steady-state the wage can be calculated using the equation

$$J(k, s) = w(k, s) - b + \beta \ast (1 - d)((1 - \psi(k))V(k, s) + \psi(k)V(k, s + 1)) - [(1 - \rho)S(k, s) + \rho S(k, s - 1))]$$

$$w(k, s) = J(k, s) + b - \beta \ast (1 - d)((1 - \psi(k))V(k, s) + \psi(k)V(k, s + 1)) - [(1 - \rho)S(k, s) + \rho S(k, s - 1))]$$

I assume for each industry there is a Constant Elasticity of Substitution (CES) aggregator of the output of different skill types with each worker employed in an industry producing one unit of industry-skill-specific output.

$$Y_k = A_k \left( \sum_s \tau_{k,s} e[k, s]^{2n - 1} \right)^{\frac{1}{n-1}}$$

I assume the production function is constant returns to scale implying $\sum_s \tau_{k,s} = 1$. The industry-skill CES parameters $\tau_{k,s}$ determine the relative marginal product of different skill levels in each industry and thus influence the relative wages. This combined with the probability of gaining skill $\phi(k)$.
determines the expected returns to staying in a given industry for a long time. The industrial productivity $A_k$ affects the relative wage across industries and therefore the relative size of different industries. Later in the quantitative exercise I will shock these productivities to induce reallocation of workers across industries. The elasticity of substitution across skills $\eta$ is an important parameter governing the response of unemployment in the model to reallocation as it controls how the relative marginal products of workers of different skill levels respond to changes in the relative supply of workers of different skill levels within an industry. So if workers move into an industry this will increase the relative supply of unskilled workers and thus decrease the marginal product of unskilled workers. If $\eta$ is large the change in marginal product will be small but if $\eta$ is close to 0 then the change in marginal product will be large and thus the value to firms of posting vacancies for these workers will fall greatly.

3.4 Household

All workers are members of the representative household. The household’s preferences over the output of each industry are given by a constant elasticity of substitution (CES) aggregator over industry output.

$$U\left(\{c_k\}_{k \in \{1, \ldots, n_k\}}\right) = \left(\sum_k \frac{1}{\omega_k^\frac{1}{\sigma-1}} \frac{c_k^{\frac{\sigma-1}{\sigma}}}{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}}$$

Where the $\omega_k$ are the CES weights and $\sigma$ is the elasticity of substitution over industry output. The profits of the firms are paid out as dividends to the
household as well as the wages of the workers.

4 Calibration

I take the model period to be a month. This allows for a reasonable frequency of churn across jobs, skill levels, employment and industries while not being so divorced from some of the data which is only available at the annual level. I set the number of industries to 4 and the number of skill levels to 2 which I label skilled and unskilled. Of the four industries, I label two high skill specificity and two low skill specificity, which will differ in their productivity $A_k$, CES production weights $\tau_{k,s}$ and skilling rate $\psi(k)$. As I will discuss later in the calibration I will use heterogeneity in the returns to industry tenure and industry mobility to differentiate between the two types of industry. High-specificity industries will feature higher returns to tenure and lower mobility than low-specificity industries. In essence, I will use industry returns as a proxy for the unobserved skill specificity of the industry. I set the number of skill levels to 2 as for each skill level I need a moment of returns to industry tenure and longer periods of tenure are more noisy due to the smaller sample size of people with long tenure. Despite allowing for only two types of industries, I still require more than two industries, as the steady state in the two-industry economy implies absolute flows from and to each industry must be equal. This would imply that the heterogeneity in flows across industries would determine the relative size of the industries. Thus in order to be able to compare the impact of a shock to same-sized industries with differential mobility I allow
for two industries for each type. Thus one type of industry can be observed to have higher mobility in steady state than the other because more workers flow between the two industries of that type than between the two industries of the other type.

I first start by fixing some parameters to values that are standard in the literature. I set the discount factor $\beta$ to 0.996 which implies an annual discount rate of approximately 5%. The parameter for the probability of losing skill while unemployed $\rho$ I take from Carrillo-Tudela and Visschers (2023) as 0.02. The model in this paper doesn’t have the idiosyncratic heterogeneity in productivity which enables Carrillo-Tudela and Visschers (2023) to explain duration dependence of unemployment which they use to calibrate this parameter. However, for the aggregates of interest in this paper, the results will not be sensitive to reasonable choices of this parameter. This is because only a small percentage of workers are unemployed each period and reasonable estimates of $\rho$ are of a similar magnitude so the changes in skill driven by skill loss while unemployed are small compared to other sources of skill change. I set the probability of death $d$ to $\frac{1}{480}$ which implies an average working life of 40 years. I set the elasticity of substitution across industry output in the household’s utility function to 4 which is within the range estimated by Broda and Weinstein (2006). I set the productivity of the matching function $\mu$ to be 0.1. As I will calibrate the vacancy posting cost $\kappa$ to match the unemployment rate, this is essentially a normalisation. If I increase $\mu$ then the calibration will increase $\kappa$ such that $\frac{\kappa}{\mu} = q(\theta_{k,s})J(k,s)$ is unchanged. This is only not the case if either $q(\theta)$ or $f(\theta)$ are truncated which a low value of $\mu$ helps avoid.
For $\eta$ the elasticity of substitution across skills in the production function, I use the value of 1. This parameter is difficult to identify as it governs the response of relative wages of skilled and unskilled workers in an industry. Hence, it requires using time series identification for which credible exogenous shocks are difficult to find. The value of 1 lies in the range that has been estimated by the literature. Dustmann, Frattini and Preston (2013) find a value of 0.6 using the response of the wages of natives to immigration in the UK. Heathcote, Storesletten and Violante (2017) find a value of 3.1 using a structural model of wages earnings and hours applied to the US. Mercan, Schoefer and Sedláček (2024) estimate an elasticity between newly hired workers at a firm and incumbent workers of 1.3 by minimizing the distance between their model and responses in the data to separation shocks. While the elasticities estimated by these papers are not directly comparable to the one in this paper as the notion of skill is different, they are all in the range of 1. More importantly they suggestive that it should be far from $\infty$ and I will show in a later section how the results are sensitive to this parameter.

I then calibrate the rest of the parameters. For the cost of vacancy posting $\kappa$ I assume it to be constant across industries and skill levels. I then calibrate it to match an aggregate unemployment rate of 4%. Then for the flow benefit of unemployment $b$ I calibrate it to match the estimates from Chodorow-Reich and Karabarbounis (2016) that the flow benefits of unemployment are 55% of wages. The rest of the parameters fall into one of two categories. First are the parameters relating to skill and production $A_k$, $\tau_{k,s}$ and $\psi(k)$. Second are the parameters relating to industry choice $\sigma_c$ and $\alpha$. To calibrate them
I will use moments of returns to career tenure, differential mobility across industries, a normalisation of average wages to 2 and an assumption that in the initial steady state, all industries have the same number of workers attached. I estimate the returns to career tenure using the NLSY79 data following Pavan (2011). I make two major changes to the specification, first I use OLS estimates rather than IV. This is because the selection effect that biases OLS occurs in the model and is informative about industry choice parameters. Secondly, I allow the returns to vary depending on 1 digit industry and I take the 25th and 75th percentiles of the estimated returns as my targets I take the estimates of industry mobility from Dvorkin (2021). I again take the 25th and 7th percentiles of the industry transition probabilities as my targets. To do this I consider the data for each industry from each period for which Dvorkin (2021) estimates a transition probability as an independent data point. The normalisation of the average wage of the employed to 2 rather than 1 is done for numerical reasons to avoid wages going negative for some guesses causing discontinuities in the returns to career tenure moments. Finally, I assume that in the initial steady state all industries have the same number of workers attached so that in the quantitative exercise I can compare how the impact of the shock varies depending on the type of industry hit.

While intuitively one might think that the returns to career tenure will primarily inform the skill parameters and the industry mobility data will primarily inform the industry choice parameters, these parameters and moments are heavily interrelated. In the case of industry returns to tenure, the OLS estimates in the data are contaminated by selection bias as workers who expe-
Table 1

<table>
<thead>
<tr>
<th>Calibrated parameters</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sectoral Productivity</td>
<td>$A_k$</td>
<td>[12.7, 15.5]</td>
</tr>
<tr>
<td>CES Production Weights</td>
<td>$\tau_{k,0}$</td>
<td>[0.11, 0.25]</td>
</tr>
<tr>
<td>Skilling Rate</td>
<td>$\psi(k)$</td>
<td>[0.024, 0.022]</td>
</tr>
<tr>
<td>Utility Cost of Switching</td>
<td>$\alpha$</td>
<td>6.0</td>
</tr>
<tr>
<td>Variance of Taste Shocks</td>
<td>$\sigma_\zeta$</td>
<td>3.0</td>
</tr>
<tr>
<td>Vacancy Posting Cost</td>
<td>$\kappa$</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Experience lower returns may be more likely to leave the industry. The model also has this selection bias as workers who have accumulated skill in an industry are less likely to leave. The degree to which mobility is selective is partially determined by $\sigma_\zeta$ the variance of the taste shocks. If $\sigma_\zeta$ is low, the staying probability will be more sensitive to the value of staying. Thus skilled workers will be much less likely to move than skilled workers, so selection bias will be high. Similarly, fixing mobility parameters, the skill and production parameters will affect the degree of mobility. The CES production weights $\tau_{k,s}$ determine the wage premium to skill and therefore the returns to staying in an industry relative to moving. The skilling rate $\psi(k)$ will play two roles, first it changes the proportion of workers who are skilled for a given mobility rate and skilled workers will move less. Secondly, it lowers the cost of moving to a new industry as workers will accumulate skill faster and so the earnings loss from moving is lower.

Another subtlety of the identification of the parameters is in the relative magnitude of $\psi(k)$, the probability of gaining skill, between the high and low skill specificity industries. A higher $\psi(k)$, all else held equal, will lead to
higher returns to tenure as workers will gain skill faster and therefore it might
be expected that the high-skill specific industry will have a higher $\psi$. This
need not be the case, however, as in order the calibration must also match
the lower mobility in the high-specificity industry. In steady state net flows
must be zero and thus in-migration must be lower. A low initial wage plus
slow skill accumulation would make the industry unattractive to workers who
would enter as unskilled. Also too high a $\psi(k)$ would lead to many workers
being skilled in the high specificity industry and given the high returns these
workers would be unlikely to leave leading to excessively low outmigration.

### 4.1 Calibrated Parameters

<table>
<thead>
<tr>
<th>Moment</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 year returns to industry tenure high specificity</td>
<td>9.0%</td>
<td>8.6%</td>
</tr>
<tr>
<td>2 year returns to industry tenure low specificity</td>
<td>3.5%</td>
<td>3.3%</td>
</tr>
<tr>
<td>5 year returns to industry tenure high specificity</td>
<td>14.9%</td>
<td>16.4%</td>
</tr>
<tr>
<td>5 year returns to industry tenure low specificity</td>
<td>6.4%</td>
<td>7.2%</td>
</tr>
<tr>
<td>Average wage</td>
<td>2.17</td>
<td>2</td>
</tr>
<tr>
<td>Transition probability away high specificity</td>
<td>5.3%</td>
<td>5.4%</td>
</tr>
<tr>
<td>Transition probability away low specificity</td>
<td>10.3%</td>
<td>11.2%</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>4.1%</td>
<td>4%</td>
</tr>
</tbody>
</table>

As can be seen in Table 2 the model in general does a good job matching
the moments of the data. The tension that stops the model from completely
matching the moments is that for the returns to tenure to be high the wage
premium must be high. However, this reduces the transition probabilities
of workers due to the high opportunity cost of losing skill. Raising $\sigma_c$ the
variance of the type 1 extreme value shocks is limited by the fact this reduces the gains from being employed. Increasing sectoral productivity would lead to an increase in the wage which is attempting to be normalised.

The estimated rates of skill accumulation of 2.4% and 2.2% per month are in line with the values assumed in Carrillo-Tudela and Visschers (2023) of 1.7% per month. The low value of the $\kappa$, the vacancy posting cost can only be understood when taking into account the productivity of the matching function which I take to be 0.1. Given the steady state vacancy fill rates cost per match ranges between 0.11 and 0.73. Compared with a marginal product of a match ranging between 1.9 and 2.26.

In order to compare the estimates of the variance of the taste shocks $\sigma_\zeta$ and utility costs of moving to those from Artuç, Chaudhuri and McLaren (2010) (ACM) and Dix-Carneiro et al. (2023) (DCPRHT) respectively, a couple adjustments must be made. First an adjustment must be made for the timing of the model as ACM is estimated at the annual level. DCPRHT propose a conversion from annual to quarterly of $\frac{\beta^4}{1-\beta^4} \frac{\beta}{1-\beta}$. Using the same formula but to go from annual to monthly would be $\frac{\beta^4}{1-\beta^4} \frac{\beta}{1-\beta}$ which equals 4.6 when using the value of $\beta$ used by ACM. Additionally as I the average wage is 2.17 rather than 1 this implies that the taste shocks should be twice as large. Combining these two adjustments with the estimate of $\sigma_\zeta$ from ACM of 1.61 gives an equivalent estimate of 16.0. This is five times as large as the estimate in this paper. On the other hand the utility cost of moving relative to the variance of the taste shocks and the wage is $\frac{6.03}{2.97 \times 2.17} = 0.94$. This is low compared to the costs estimated in DCPRHT who allows the cost to vary for every source-destination.
pair and finds values ranging between 0 and 3.43.

A potential driver of this difference is the presence of industry-specific skill. This acts as an incentive to stay within an industry that is absent from ACM and DCRPHT. Thus it is unsurprising that the mobility cost estimate is lower and thus a smaller variance in the taste shock is required. Secondly in ACM the estimates of $\sigma_\zeta$ vary depending on the value chose of $\beta$ with lower $\beta$ implying a lower value. Since the $\beta$ I chose is lower this may be driving some of the difference in this parameter.

4.2 Earnings Losses from Displacement

In order to validate the model, I compare the earnings losses from displacement in the model to the data by industry stayers and leavers. This is the moment that Huckfeldt (2022) targets in the calibration of his model. As I target instead the industry returns to tenure this is a useful check in two senses. First, it alleviates concerns that the results might be driven by specific features of the industry tenure moments. One potential concern is that the selection in the model may not be of a similar magnitude to the selection in the data and thus the calibration may over or understate the underlying returns to tenure. Secondly, this is a moment informative about the micro costs of reallocation. For the results of the model for the macro costs of reallocation to be credible, the micro costs must be realistic.

To calculate the earnings losses from displacement I need to take a stand on what displacement is in the model. In the data, workers are considered displaced if they lose their job for reasons of slack work, plant closings, and
abolished jobs which are considered exogenous to the worker. In the model all job destruction is considered exogenous to the worker, however the level of job destruction is high to allow for ‘job-to-job’ transitions in the model without directly modeling them. For this reason, a large number of workers who lose their job in a period will find a new job in the same period. So if I were to label all workers who separate at the beginning of a period as displaced, the earnings losses of stayers would be small as it would be dominated by these ‘job-to-job’ transitions. Therefore I define displacement as a worker who separates from their job and is unemployed for at least one period. I then define an industry stayer as a worker who is next employed in the same industry as the job from which they are displaced and an industry leaver as a worker who is next employed in a different industry.

To construct the comparison group I use workers who are not separated from their job in the period. I start with the steady state distribution of workers across industries and skill levels. I then iterate forward the distributions of workers who are both displaced and not displaced. This gives me the full time path of these distributions without simulation error. I then use this distribution and the wages to calculate the average monthly earnings of all three groups. Then to compare to the data I aggregate up to an annual frequency.

I plot the results in Figure 3. Comparing this to the data on earnings losses from Huckfeldt (2022) the model does a good job of matching the earnings losses of industry stayers. The losses on impact are very close to those of the data, with around 20% for stayers and 40% for leavers. Additionally, the model also does a good job of matching the dynamics of earnings losses over
time. The earnings jump up in the first year after the displacement and then slowly recover over time. Given both the initial impact and the dynamics are untargeted in the calibration, this is a strong validation of the model’s ability to capture the micro level costs to workers of reallocation.

Figure 3: Comparison of Employment Depending Industries Shocked

5 Quantitative Experiment

In order to understand the effect of reallocation on aggregate unemployment in the model I study the response to a shock to the productivity of two industries in the economy. One industry receives a positive productivity shock and the other a negative productivity shock. The shock takes the form of an
unanticipated MIT shock which takes effect in a linear manner over a decade. I determine the shock size by finding the negative shock to a high-skill specific industry and positive shock to the other high-skill specific industry that leads to the same steady state employment as the initial steady state and a change in industry shares in line with decadal changes in industry shares. I then take the same sized negative shock and solve for the positive shock that leads to the same steady state employment for all other combinations of industry types getting shocks.

I plot the results for unemployment in Figure 4. There are two main takeaways from this figure. First, is that the reallocation shock can lead to a substantial increase in unemployment in this model. The lowest trough in unemployment is 0.8 percentage points below steady state, which is a 17% increase from steady state unemployment. Additionally, the unemployment response is highly persistent, with the recovery taking a decade to complete. It is important to note that unlike Chodorow-Reich and Wieland (2020) this effect does not require a coinciding negative aggregate demand shock nor downward nominal wage rigidity.

Secondly, the impact of the reallocation shock is heterogeneous in both magnitude as well as dynamics with reallocation involving high skill specificity industries having larger effects. In particular if the growing industry is high specificity this leads to an persistent rise in unemployment. This is because skill accumulation is slower in the high specificity industry and so it takes a long time to reach the new steady state level of skilled workers in that industry. The size of the shock in the very short run is larger if the shrinking industry is
high skill specificity. This is due to the skilled workers in the high specificity industry being less willing to move and therefore exposed to the negative shock to their industry.

These effects are driven by the dynamics of marginal productivity of different workers. To show this I plot in Figure 5 the dynamics of workers’ marginal product to the shock that reallocates between the high skill specificity industries. The marginal products of the skilled workers in the shrinking industry and the unskilled workers in the growing industry both fall in response to the shock before slowly recovering. This happens to the skilled workers in the shrinking industry due to both the decline in productivity but also the exit of unskilled workers from the industry. This drives down the marginal product of skilled workers as the relative supply of skilled workers increases. While for the unskilled workers in the growing industry, the productivity shock is positive for their marginal product however this is dominated by the negative effect entry of workers into the industry. For unskilled workers in the shrinking industry and skilled in the growing their marginal product increases driven primarily by their relative share of industry employment falling.

5.1 The Dynamics of Industry Transitions

An important feature of the model is that workers move across industries. In Figure 6 I plot how the absolute flows of workers out of industries evolves in response to the shock discussed above which reallocates from a high skill specificity industry to the other one. In order to illustrate the change from the initial steady state the graph begins the month before the shock is realised. I
Figure 4: Comparison of Employment Depending Industries Shocked

All lines are in response to productivity shock to the two industries. One hit positively and one negatively. The shocks are chosen to match decadal changes in industry shares and such that the new steady state features the same level of employment as the initial steady state.

The first thing to notice is that the absolute flows of unskilled workers is always higher than the flows of skilled workers. This is because skilled workers face a larger opportunity cost of leaving in the form of losing their accumulated skill.

In the first panel are the dynamics of moves out of the industry receiving the positive shock. For both skilled and unskilled workers the number of
The shock is to the two high-specificity industries. One hit positively and one negatively. The shocks are chosen to match decadal changes in industry shares and such that the new steady state features the same level of employment as the initial steady state.

flows drops on impact as the wages of this industry respond to the shock. For unskilled workers this quickly reverses as wages decline due to the entry of unskilled workers and the substitutability across skills in production as described previously. The flows of skilled workers declines due to the rise in the opportunity cost driven by the increase in the wages paid to skilled workers in this industry. In the longer run the number of flows of skilled
workers increases as the skill premium declines and number of workers in the industry rise.

In the middle panel are the dynamics for the industry receiving the negative shock. In the short run the results are exactly the opposite of the case of the growing industry. The number of flows of both skilled and unskilled rise due to the direct effect of the shock on wages. The outflow of skilled workers then slowly declines as the skilled to unskilled ratio in the industry returns to steady state and the absolute size of industry declines. The outflows of unskilled workers similar to before quickly jumps back due to the change in the skill ratio. After this it begins to rise again as the value of the being a unskilled in any industry falls due to the increased number of unskilled workers as skilled workers leave the shrinking industry. Thus it begins falling again as the ratio converges to the new steady state. These dynamics are a consequence of modelling industry choice as a discrete choice subject to idiosyncratic type I extreme value taste shocks. When the value of all options falls, the taste shocks become more important pushing towards increased moves across industries. For the low skill specificity industries the outflows of unskilled workers rises and then falls along with the general rise and fall of being unskilled in any sector. The outflows of skilled workers moves little.

Carrillo-Tudela and Visschers (2023) argue that this increase in gross moves is inconsistent with the data. They proposed an alternative formulation of search across industries in which search is costly. Thus when the values of being unskilled fall moves gross moves fall.
5.2 The Role of Substitutability Between Skills

The importance of changing relative supplies of different skills to the effects points to $\eta$ the elasticity of substitution between workers of different skill levels as a key parameter in the model to generate unemployment in response to reallocation. In this subsection, I illustrate the impact of this parameter on the results of the model as well as the mechanism through which it operates. I rerun the counterfactual experiment with different values of $\eta$ and plot the
results in Figure 7.

Figure 7: Comparison of Employment depending of $\eta$

All lines are in response to productivity shock to the two high-specificity industries. One hit positively and one negatively. I resolve the steady state for each $\eta$. The shocks are chosen to match decadal changes in industry shares and such that the new steady state features the same level of employment as the initial steady state.

As can be seen in the figure, the impact of the demand shock on employment is decreasing in $\eta$ and the effect is substantial. Going from an $\eta$ of 0.5 to an $\eta$ of 10 reduces the size of the unemployment response by over 50% The higher the elasticity of substitution the less responsive the relative marginal products of different skill levels are to changes in the ratio of workers of different skill levels. So when unskilled workers leave the industry with the negative productivity shock, the relative marginal product of skilled workers falls by less the higher $\eta$. Similarly the relative marginal product of unskilled workers in the industry with the positive productivity shock fall by less the higher $\eta$ as workers enter the industry. As these are the locations where most of the unemployment occurs, the more their marginal products fall the more unemployment there is.
This is the important difference from Carrillo-Tudela and Visschers (2023). In their model workers have idiosyncratic productivity for the sector they are in which accumulates as well as having stochastic variation. However, in their model workers with different productivities are perfect substitutes so when there is a demand shock to a sector the marginal product of workers of all skills will rise no matter the skill distribution. Thus the value to a firm of the low productivity workers still increases when workers without industry specific skill enter. So firms post enough vacancies to absorb these incoming workers. So despite their model having specific skill, reallocation does not increase unemployment.

5.3 Mismatch Unemployment

Şahin et al. (2014) proposed an index of mismatch $M_t$ defined as below.

$$M_t = 1 - \sum_k \left( \frac{\phi_{k,t}}{\bar{\phi}_t} \right) \left( \frac{v_{k,t}}{v_t} \right)^\eta \left( \frac{u_{k,t}}{u_t} \right)^{1-\eta} \tag{4}$$

Where $\phi_{k,t}$ is the industry-level matching efficiency, which in this paper is constant $\mu$. $\bar{\phi}_t$ is the economy-wide matching efficiency, which is also constant at $\mu$. $v_t$ and $u_t$ are the aggregate number of vacancies and unemployed workers.

Şahin et al. (2014) found that this index could explain only one-third of the rise in unemployment during the great recession. I calculate this index for the model economy in response to reallocation between the two high-skill specificity industries and plot it in Figure 8. In the left hand side figure is
the dynamics of the Şahin et al. (2014) index. It does rise in response to the shock however it remains small peaking at just over 1% of total unemployment being ascribed to mismatch. As the shock generates additional unemployment of around 17% the index correctly shows that the unemployment generated is not due to mismatch across industries.

The mismatch in this model is between workers of different skill levels within an industry. Thus the correct index to use is one that accounts for mismatch across skills.

$$M_t = 1 - \sum_k \sum_s \left( \frac{\phi_{k,s,t}}{\bar{\phi}_t} \right) \left( \frac{v_{k,s,t}}{v_t} \right)^\eta \left( \frac{u_{k,s,t}}{u_t} \right)^{1-\eta}$$  

(5)

I plot the dynamics of this index in response to the shock in the right hand side of Figure 8. Here the measure increases by much more than in the previous case, peaking at almost 18% of total unemployment. However given it starts from a much higher level of 14% this rise still doesn’t fully explain the rise in unemployment.

### 6 Random Search

The assumption of directed search is strong, implicitly assuming full information on the skills of workers. In this section, I consider the other extreme of random search where firms meet workers at random. To avoid adding additional complications from learning like those considered in Baley, Figueiredo and Ulbricht (2022) I assume that workers know their own skills and it is re-
All lines are in response to productivity shock to the two high-specificity industries. One hit positively and one negatively. The mismatch indexes are as described in the text.

vealed to firms upon matching. This changes the free entry condition in the model to be

\[ \kappa = q(\theta_k)\mathbb{E}_s [J(k, s)] \]

Where \( \theta_k \) is the labor market tightness of industry \( k \) defined as the vacancies posted by firms in that industry divided by the number of unemployed workers summing over all skill levels. The expectation of \( J(k, s) \) is taken with respect to the distribution of skill levels among unemployed workers. I assume that workers of all skill levels are equally likely to find a match. Thus in the case
of two skill levels where the share of workers who are unskilled is denoted $\chi_k$
this can be expressed as

$$E_s [J(k, s)] = \chi_k J(k, 0) + (1 - \chi_k) J(k, 1)$$

So as in equilibrium $J(k, 1) > J(k, 0)$ due to calibration targeting wage growth, an increase in the $\chi_k$ will decrease $E_s [J(k, s)]$. Thus as more workers in the unemployed pool are unskilled the benefit of posting a vacancy decreases.

I then recalibrate the model to match the same moments as in the directed search model. Then I feed in the same productivity shocks as in the directed search model and compare the results in Figure 9. In the directed search model there is no large decrease in employment for any of the shocks. In fact for a couple of the shocks employment goes above the steady state level and converges back to steady state from above.

The reason for this is that the effect on relative marginal products cancel out in the vacancy posting decision. As the relative supply of unskilled workers increases, their marginal product decreases while the marginal product of skilled workers increases. However, due to random search firms can’t direct their vacancies by skill and so the first part of the change in marginal products decreases $E_s [J(k, s)]$ but the second increases it.

There is also an effect from changes in the distribution of skill among the unemployed. Workers who enter from other industries enter as unskilled and unemployed. This decreases $E_s [J(k, s)]$ potentially leading to lower vacancy
Figure 9: Comparison of Unemployment Response

![Graphs showing the comparison of unemployment response.](image)

(a) Random Search

(b) Directed Search

All lines are in response to productivity shock to the two industries. One hit positively and one negatively. The shocks are chosen to match decadal changes in industry shares and such that the new steady state features the same level of employment as the initial steady state.

posting. However, this effect on the distribution of skill among the unemployed does not have a large persistent impact on vacancy posting. If firms decrease vacancy posting then fewer skilled workers who become separated with regain employment. As the share of skilled workers in employment is relatively large and the separation rate $\delta$ high this means that small changes in vacancy posting relative to unemployment will have a large impact on the distribution of skill among the unemployed.

Thus the ability of firms to distinguish between skilled and unskilled workers is key to the finding on a negative effect of reallocation on unemployment.
Given that firms do observe industry tenure as well as job titles and responsibilities as well as the slow rate of skill accumulation from the calibration, directed search may well describe the labor market better in this particular setting.

7 Conclusion

This paper argues that the reallocation of labour demand can have consequences for aggregate unemployment. This result comes from allowing for a realistic structure of substitutability between workers of different skill levels. When different skill levels are not perfect substitutes the demand for unskilled workers will be lower in the transition than in steady state. As this is also where there is a greater supply of workers during the transition this can lead to transitory unemployment.

That substitutability between workers is important for the response to shocks may also apply to other cases. Many modern macro models assume the marginal product of a match is independent of the distribution of matches in the economy for tractability. So there is a need to better understand when this powerful assumption is a good approximation to the real world.
References


A Appendix

A.1 Chodorow-Reich and Wieland Replication

In Chodorow-Reich and Wieland (2020), the authors present evidence that the impact of the reallocation of labor demand on unemployment varies with the business cycle. They use data from the Quarterly Census of Employment and Wages (QCEW) to construct a measure of reallocation they propose.

\[
R_{s,t,t+j} = \frac{12}{j} \sum_{i} w_{s,i,t} \frac{|g_{s,i,t,t+j} - g_{s,t,t+j}|}{g_{s,t,t+j}}
\]

Where \( s \) is the county, \( i \) indexes industries, \( t \) and \( t + j \) are the periods that bracket the time over which reallocation. \( g_{s,i,t,t+j} \) is the employment growth of industry \( i \) in \( s \) from \( t \) to \( t + j \). While \( g_{s,t,t+j} \) is the employment growth of county \( s \) from \( t \) to \( t + j \) across all industries. This is the weighted average absolute deviation of industry growth rates within a county. To see how this measure captures the concept of reallocation, consider first the case in which employment is constant across all industries. Then this measure will be at its minimum of 0. Otherwise, if there is one industry that is growing in employment and one that is declining, both at the same rate, then the measure will be strictly positive and increasing in that rate. The choice of absolute value norm over the squared norm is motivated to avoid giving too much influence measurement error in employment of small industries which could result in large positive or negative growth rates.
The specification that Chodorow-Reich and Wieland (2020) use is as follows

\[ \Delta u_{s,t,t+j} = \beta R_{s,t,t+j} + \theta R_{s,t,t+j} \times \text{Recession and Recovery} + \gamma PD_{s,t,t+j} + \delta_t + \epsilon_{s,t,t+j} \]

Where \( \Delta u_{s,t,t+j} \) is the change in unemployment rate, \( \delta_t \) is a time fixed effect, and Recession and Recovery is a dummy for phase of the cycle (recession and recovery or boom) \( PD_{s,t,t+j} \) is the usual Bartik predicted demand instrument defined as

\[ PD_{s,t,t+j} = \frac{1}{2} \sum_i w_{s,i,t}(1 + g_{i,t+j}) \]

The time fixed effect keeps the comparisons made in the regression to within the same aggregate state of the business cycle. The Bartik predicted demand instrument is used to control for the shocks to the level of labor demand in a county from movements in the aggregate.

The key element of the identification strategy is that the authors instrument for the local level of reallocation with the level of reallocation predicted by the national industry growth rates and local industry employment shares. This instrument takes the form

\[ R_{s,t,t+j} = \frac{12}{j} \sum_i w_{s,i,t} \left| \frac{g_{i,t,t+j} - g_{t,t+j}}{g_{t,t+j}} \right| \]
Where the growth rates not indexed by $s$ are the national growth rates. There are several issues with this empirical design which I will discuss in the following order. First, the qcew data at the county industry level is has issues with discontinuities which leads to overestimation of reallocation. Secondly, the instrument is biased due to larger industries having systematically smaller amounts of reallocation. Thirdly, some of the controls used are inappropriate.

A.1.1 Discontinuities in QCEW Data

Despite the administrative nature of the QCEW data, there are issues which are highly relevant to the measurement of reallocation. There are many discontinuities in the data. When a discontinuity occurs, this is measured as a large amount of reallocation whether the discontinuity is up or down. These observations thus have a large influence on the first stage of the regression. The regression procedure will attempt to predict the observed reallocation due to discontinuities. However, as these discontinuities are not related to the underlying economic conditions, these predictions will be spurious and the resulting fitted values used in the reduced form will be partially spurious. This is one potential explanation for the coefficient in the first stage being a third smaller in expansions compared to recessions and recoveries.

One potential source of discontinuities is strikes. Striking workers are not recorded as being employed leading to a large decline in local employment within an industry for a short period of time. As strikes are temporary work stoppages and not an end of the employment relationship they lead to erroneous measurements of reallocation. However this only explains temporary
discontinuities which then return to the previous level of employment. Large one time discontinuities could be explained by the reclassification of establishments across industries which leads to discontinuities in employment for both the old and the new industries. While the reclassification of an establishment may be driven by shifts in the activities of the establishment, those establishments that are reclassified are likely those that are closest to the boundaries between industries so the tasks of the workers are likely to remain similar. This is supported by the reclassification leading to similar changes in employment in the old and new industries indicating the establishment retains most of its employees. Thus the large measured reallocation induced by a reclassification does not reflect a large change in the required skills demanded by the establishment so is spurious.

A.1.2 Instrument Bias

Borusyak and Hull (2022) note that their proof of asymptotic unbiasedness of formula based instruments does not apply to non linear instruments. In this section I will show how a simpler version of the instrument used by Chodorow-Reich and Wieland (2020) is biased and how to correct for this bias following the logic of Borusyak and Hull (2022). Consider the following definition of reallocation.

\[
R_{s,t,t+j} = \frac{12}{j} \sum_{i} w_{s,i,t} |g_{s,i,t,t+j} - g_{o,t,t+j}|
\]
Where the corresponding instrument is

\[ R_{s,t,t+j} = \frac{12}{j} \sum_{i}^{l} w_{s,i,t} |g_{i,t,t+j} - g_{t,t+j}| \]

The condition for the instrument to be unbiased is that the expected value conditional on the weights is constant across locations.

\[ E[R_{s,t,t+j}|w] = c \]

\[ E[R_{s,t,t+j}|w] = \frac{12}{j} \sum_{i}^{l} E[w_{s,i,t}|g_{i,t,t+j} - g_{t,t+j}|w] \]

\[ = \frac{12}{j} \sum_{i}^{l} E[w_{s,i,t}(1 - w_{i,t,t+j})g_{i,t,t+j} - \sum_{j \neq i} w_{j,t}g_{j,t,t+j}|w] \]

\[ = \frac{12}{j} \sum_{i}^{l} w_{s,i,t}(1 - w_{i,t,t+j})E[|g_{i,t,t+j} - \sum_{j \neq i} \frac{w_{j,t}}{(1 - w_{i,t,t+j})g_{j,t,t+j}}|w] \]

Thus even under the assumption that all the growth rates are independent and identically distributed the expected instrument will not be constant across
locations. Instead it will be proportional to $\sum_i^I w_{s,i,t}(1 - w_{i,t,t+j})$. Larger industries will generate smaller amounts of reallocation because they change the average by more. Thus locations which are more exposed to industries with larger employment shares will have smaller expected values of the instrument.

The solution proposed by Borusyak and Hull (2022) is to control for this differential exposure to the instrument by including the term $\sum_i^I w_{s,i,t}(1 - w_{i,t,t+j})$ in the first stage regression.

### A.1.3 Inappropriate Controls

Chodorow-Reich and Wieland (2020) include a number of controls in their regression however I will focus on lagged population growth and lagged employment growth. The inclusion of these controls is inappropriate because the dependent variable is the change in unemployment rate. Since the change in the unemployment rate is going to be driven by changes in the size of the labor force and changes in the number of workers employed these two controls will together be highly correlated with lagged unemployment changes. However, including lagged unemployment change is not appropriate due to the lagged dependent variable problem. Any omitted variables that affect $u_{s,t}$ will be correlated with lagged unemployment change as well as appearing in the true error term. This introduces bias into the estimation of all coefficients in the regression.